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A Self-Contained Standard Harmonic Wave-Meter

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VII. *A Self-contained Standard Harmonic Wave-meter.*

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1. Summary and Introduction.

Summary.

Following a brief introductory survey of the various methods of measuring radio-frequency, the principles of a highly accurate standard harmonic wave-meter are developed.

The essential principle is an arrangement of three-electrode valves, known as a multi-vibrator, the invention of H. Abraham and E. Bloch. The arrangement produces a discontinuous wave, the frequency of which may be adjusted to have any value within very wide limits. When a current of such wave form acts by induction on another circuit, this circuit receives what may be considered as a series of electrical blows at equal intervals of time. If the circuit operated upon is highly resonant, then when its resonant frequency is set so as to be an integral multiple of the frequency of the blows, a large oscillatory current is built up, of a persistent kind and of a frequency whose ratio to that of the blows is quite exact.

In the apparatus developed and described there are two such multivibrators; one of these has a frequency of 1 kilocycle per second and the other has a frequency which is usually 20 kilocycles per second. The most important feature of the wave-meter is the means whereby the frequencies are kept constant. The two multivibrators are kept in harmonic synchronism with each other by the help of the reinforced twentieth harmonic of the low-frequency multivibrator. This is accomplished by the help of a special selector circuit and amplifying valve arrangement, tuned to a frequency of 20 kilocycles per second. The two multivibrators are further both controlled in frequency by means of an electrically maintained tuning-fork, of frequency of 1 kilocycle per second. This complete system therefore forms a source of electrical impulses of extremely steady and accurately known frequencies.

The impulses from either multivibrator can operate on a selector circuit, consisting of a standard variable condenser of special design and a set of six inductance coils of small damping decrement. By this means two complete series of harmonics may be selected,

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covering a range between 10 and 1,200 kilocycles per second. The frequencies produced do not depend upon the constancy of any electrical circuit. A selective amplifier serves to amplify any selected harmonic and by interference with an external source this latter may be set to exact synchronism with the harmonic.

The paper is divided into seven main sections as follows :—

(1) Introductory.

(2) to (4) Principles of the multivibrator and experiments therewith to prove its suitability for the purpose in view.

(5) Fundamental principles governing the design of the wave-meter.

(6) General outline of the principal features of the construction, including the design of a condenser to give a uniform frequency scale.

(7) Methods of using the wave-meter and the possibilities connected therewith.

The paper concludes with a short account of the precautions necessary to ensure certainty in using the wave-meter and some conclusions are given.

Introduction.

The measurement of the frequency and wave-length of electrical oscillations and waves has attracted the attention of many workers since the discovery of the oscillatory character of the discharge from a Leyden jar. The various methods of measurement adopted fall naturally into two classes : (1) Indirect methods, (2) direct methods.

(1) The indirect methods used have been many, but they fall mainly into two sub-divisions :—

(*a*) Those depending upon inductance and capacity in an oscillatory circuit and upon the exact adjustment of one or both of these so as to bring the circuit into the condition of electrical resonance or synchronism with the circuit under measurement.

(*b*) Those depending upon the ratio of currents in two branch circuits, the relative impedances of which vary with frequency. Instruments of this kind can be made direct reading in frequency or wave-length.

(2) The direct methods have been used to measure either frequency or wave-length, depending upon the region investigated ; amongst the various methods which have been used may be mentioned :—

(*a*) High-frequency alternators.

(*b*) Photography of luminous discharges after reflection from a revolving mirror.

(*c*) Harmonic methods.

In the case of very high frequencies many measurements have been made by the help of stationary waves on wires. The wave-length is determined as a direct measurement and the frequency is deduced by calculation, using the accepted value of v.

Though of much interest, space forbids more than a few passing comments on these various methods of measurement.

The indirect methods depending upon resonance in an oscillatory circuit form the basis of the great majority of wave-meters constructed as portable instruments. The main advantages are ease and certainty of operation and adaptability to the end in view. The accuracy also compares favourably with other electrical instruments whose indications are read off on a scale. The chief disadvantage of such wave-meters is the liability of the calibration to alter with time; this is mainly due to changes in the variable condensers commonly used in such instruments. For certain precise work also the setting of the condenser to the condition of synchronism cannot always be made with sufficient accuracy. There are, however, null methods whereby the condition of resonance may be indicated, and these are more accurate; they have not, as yet, been developed so as to be incorporated into a self-contained wave-meter.

The introduction of the self-generating wave-meter using a triode valve has removed the difficulty of accurate setting to resonance, but the effects of the valve and its associated circuits on the setting for resonance or synchronism may be serious.

Both of these types of wave-meter, however, serve very useful purposes, and for a great deal of work they cannot be displaced by any superior means of measuring frequency or wave-length at present.

By the use of carefully constructed standard inductance coils or loops and standard condensers, a standard of radio frequency accurate to one or two parts in a thousand can be established, but ultimately recourse must be had to a more absolute and direct method of measurement. The increasingly severe requirements for steadiness and accuracy of frequency of transmissions, and of tuning of receiving apparatus, render necessary a greater degree of accuracy and permanence in the wave-meters or other measuring devices than can be achieved in an instrument of the usual oscillatory circuit pattern.

The attention of many workers has therefore been turned to the direct methods of measuring radio frequency. Of these, the high-frequency alternator is the most direct of all, and in those transmitting stations equipped with steadily running high-frequency alternators nothing more is needed than a means of measuring the rate of revolution of the alternator shaft. The limitations of this method are obvious and need not be considered further.

The method of photography of luminous discharges, whilst fulfilling a useful purpose in the history of the development of frequency measurement, is complicated and troublesome to operate. It is further limited entirely to laboratory conditions; the results also are not directly known and are not very accurate on a single observation.

Much attention has been concentrated on harmonic methods of building up or down in frequency, so as to obtain a frequency or series of frequencies which are exact multiples or sub-multiples of some standard frequency which is known or can be measured.

Amongst those who have worked in this field may be mentioned SIMON and BARKHAUSEN (1), of the 'Physikalisch-Technische Reichsanstalt,' who developed the theory of harmonics in the Poulsen arc and showed what conditions governed the suppression or augmentation of harmonics, as desired. LINDEMANN (2) made use of these principles in the calibration of wave-meters ; H. DIESSELHORST and R. SCHMIDT (3) also have shown Lissajous figures of harmonics from a Poulsen arc by the help of a cathode-ray tube.

In 1919 MANDELSTAM, in working with buzzer-excited oscillations, observed that an augmentation of the current in the radio-frequency circuit was obtained when the tuning in this circuit brought the radio-frequency into exact harmonic synchronisation with the telephonic frequency of interruption of the buzzer current. There is also a small amount of control exerted by the buzzer-frequency on the radio-frequency when the latter is brought into almost perfect synchronisation with the former. TYKOCINSKI-TYKOCINER (4) carried out a careful investigation on these lines, using a buzzer which gave regular interruptions of telephonic frequency.

The steadiness of operation and adaptability of the three-electrode valve has, however, raised the level of accuracy attainable by harmonic methods to a much higher plane. J. S. TOWNSEND and J. H. MORRELL (5) experimented, at the short-wave end, on the harmonics and overtones of solenoids, using a Lecher wire system and building downwards in frequency harmonically. In a recent Japanese publication also, by AKIRA TSUBOUCHI (6), harmonic standardisation of wave-meters has been accomplished by using a standard circuit consisting of a single-loop inductance and standard condenser of Bureau of Standards pattern. Still more recently G. W. PIERCE (7) has utilised the mechanical resonances of piezo-electric crystal resonators (8) and crystal oscillators to obtain harmonic series of frequencies of known values in terms of the fundamental standard frequencies of the oscillators.

But the multivibrator of H. ABRAHAM and E. BLOCH (9) appears to be the most valuable source of harmonics yet devised for the purpose of producing known radio frequencies and possessed of the necessary steadiness and constancy.

This device forms the principle of the wave-meter about to be described, but before proceeding to this description, a very brief outline of the multivibrator principle may be permitted here in order to make the operation of the wave-meter clear and intelligible. (A complete description of the multivibrator is, of course, given in the paper by H. ABRAHAM and E. BLOCH (9), mentioned above.)

2. PRINCIPLE OF THE ABRAHAM-BLOCH MULTIVIBRATOR.

The scheme of connections is shown in fig. 1, which is symmetrical except for the connections at x in the grid leak path of valve V_2. An inducing coil is connected to these terminals for the purpose of applying the electrical impulses to an oscillatory circuit.

Referring to fig. 1, r_1 and r_2 are two resistances of about 50,000 ohms each in the main anode circuits. The points P_1 and P_2 are connected to the grids G_2 and G_1 respectively through adjustable condensers C_1 and C_2. The grids G_1 and G_2 are connected through leak resistances R_1 and R_2 respectively to the filaments. These resistances have the value of about 75,000 ohms each.

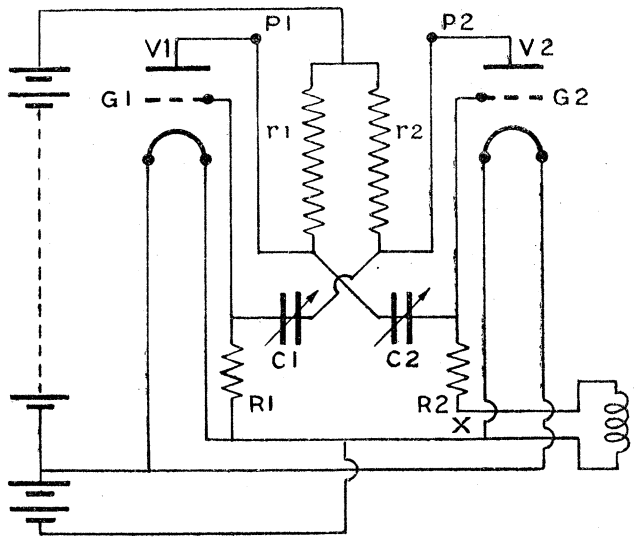

FIG. 1.—Abraham-Bloch multivibrator.

The operation is most easily followed by reference to diagram fig. 2, reproduced from actual photographs taken at the National Physical Laboratory on an Einthoven string galvanometer, of a multivibrator operating at a fundamental frequency of about 20 cycles per second.

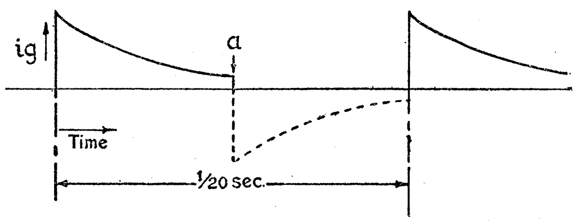

FIG. 2.—Grid-filament current in multivibrator.

The curves give current variations in the grid coil on X. The voltage variations between grid and filament will be approximately similar to the current variations. Assume the apparatus to be operating in its steady cyclic state and consider the sequence of events immediately following an inversion at the instant "*a*." The voltage of G_2 has become strongly negative and has caused the anode current in valve V_2 to suddenly become zero. This negative voltage diminishes towards zero by reason of leakage through R_2, of the charge on the condenser C_2 and on G_2. When the voltage has become small, but before reaching zero value, anode current begins to flow again in valve V_2; this causes a diminution of anode voltage at the point P_2; this diminution is transmitted *via* condenser C_1 to grid G_1. The result of this fall in potential of G_1 is to cause a decrease in anode current of V_2, V_1 thus increasing the voltage at the point P_1. The consequent rise in grid potential of G_2 accelerates the increasing anode current in V_2. Finally the potential of grid G_1 becomes sufficiently negative as a result of the increased potential fall on r_2 to reduce the anode current in V_1 to zero. The cycle outlined above now recommences, and may be traced by interchanging the suffixes 2 and 1 wherever they occur above. When the change-over occurs—at such an instant as "*a*"—it is to all intents and purposes quite instantaneous, even in terms of a period of oscillation of 1/1000 second.

The period of the cycle is roughly equal to the sum $C_1 R_1 + C_2 R_2$; thus in the actual multivibrator constructed, $C_1 = C_2 = 0 \cdot 0062$ μF and $R_1 = R_2 = 75{,}000$ ohms, giving a calculated value, on this basis, of 1/930 sec. for the period, the actual value of which was 1/1000 sec.

The resistances r_1 and r_2 have not been assumed to have any effect on the frequency at which the apparatus works, but there is little doubt that they do govern the behaviour to a large extent. In the original description of the apparatus by H. ABRAHAM, r_1 and r_2 were inductive coils wound with high resistance wire for the case of a multivibrator having a fundamental frequency of 1,000 cycles per second. It is, however, not necessary for the resistances r_1 and r_2 to be inductive. For a multivibrator having a fundamental of the order of 20,000 cycles per second, the coils r_1 and r_2 are wound with copper wire and the frequency is not very dependent upon the values of C and R.

Experiments with the Multivibrator.

A number of experiments on the multivibrator have been made from time to time by the author with a view to testing its suitability for incorporation into a self-contained and direct-reading standard harmonic wave-meter. The more interesting of these will now be described.

(*a*) *Variations in Frequency of the Fundamental.*

Observations were made on the variations in frequency of the fundamental in the absence of control when anode voltage, filament brightness, etc., were varied. The observations were made by counting beats against a standard tuning-fork of a nominal frequency of 1,000 vibrations per second. It was only possible, by this means, to observe the effects over a narrow range of variation, on account of difficulties in counting beats when these are at a greater rate than four or five per second.

Observations on variations in frequency due to change of anode voltage are given in Table I.

TABLE I.

Anode Battery Voltage.	Beats per second (Multivibrator — Tuning-Fork).
79·0	+ 2·6
80·0	0·0
81·0	— 2·1

It is seen that a change of +1 per cent. in voltage of the anode battery produces a change in frequency of approximately — 2 parts in a thousand.

Experiments on the effect of altering the filament current were then made and gave the results shown in Table II.

TABLE II.

Filament Current (through both valves).	Beats per second (Multivibrator — Tuning-Fork).
1·25	+ 2·5
1·28	— 0·5
1·30	— 2·3
1·315	— 3·4

Thus the change in frequency due to a + 1 per cent. change in current is about — 1·1 parts in 1,000.

Measurements made, of the drift in frequency, after first switching on showed that after ten minutes the frequency became steady to within a very few parts in ten thousand. Other tests, such as bringing the hand near the apparatus, or altering the coupling between the output coil and an external circuit, were found to cause changes in frequency of the order of 1 or 2 parts in 1,000. When everything is in a steady state, however, the frequency will remain constant to within a few parts in 10,000 for considerable periods of time.

In order to overcome the variations in frequency a system of control was devised, using a valve-maintained tuning-fork to hold the multivibrator in synchronism. This feature has been incorporated into the wave-meter in its final form.

(b) *Wave Shape of Harmonic Resonance Curves.*

A multivibrator so controlled was set up and measurements were made on the intensity of current, under its influence, in an oscillatory circuit, consisting of a variable condenser reading to 2,000 $\mu\mu$F, and an inductance coil of about 20 millih. inductance and between 7 and 8 ohms effective resistance at the radio-frequencies concerned. Preliminary experiments using a Duddell thermogalvanometer in series in the circuit

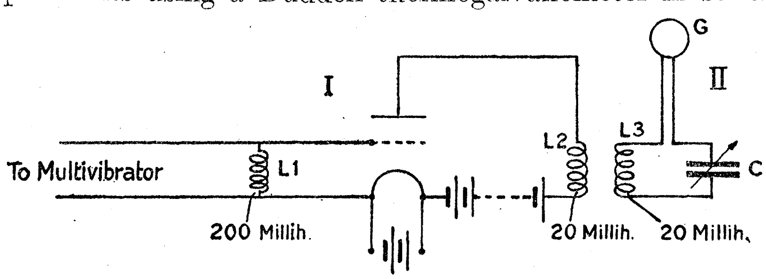

FIG. 3.—Measurement of current induced in a selector circuit.

showed that, in order to obtain sufficient deflection, a heater of such resistance would be required as to spoil the sharpness of the resonances. A single stage of amplification was accordingly used on the multivibrator; the circuit was as in fig. 3. The grid filament circuit of the valve was connected to the terminals of an inductance coil of 200 millih. carrying the impulse current from the multivibrator. In the anode circuit was a coil of 20 millih., which induced into the variable selector circuit II containing the Duddell thermogalvanometer G. A series of a great number of observations was made, at selected readings on the condenser, of the galvanometer deflections, covering a range of about 20 harmonics. The induced current in the selector circuit has been plotted in fig. 4 against condenser reading for two series of observations.

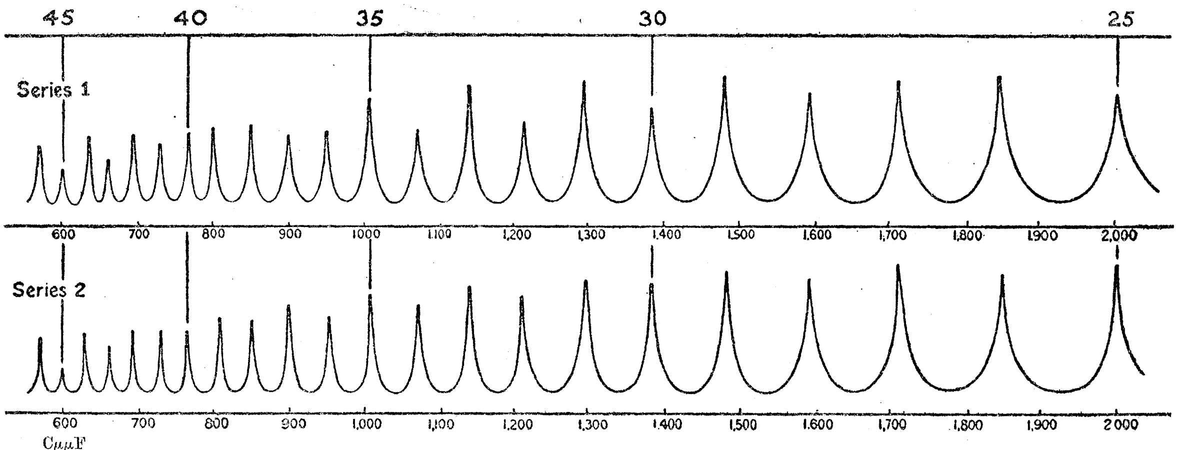

Fig. 4.—Current induced in selector circuit.

These interesting curves show the extraordinary sharpness of resonance attained at each reinforcement of a harmonic. An interesting feature of the peaks is that the tips of them fall on two curves, one joining the odd harmonics and the other the even ones. It will be observed that in both series in the region of the 45th harmonic, the even harmonics are stronger than the odd ones, whereas in the region of the 32nd harmonic the position is reversed. The curves joining the peaks are not always the same; there is usually an undulation on each curve, and these undulations intersect one another at certain regions. Considerable variations in the intensities of the harmonics in different regions can be produced by altering the setting of the multivibrator condensers, keeping the variations within the limits of synchronisation by the fork. There is little doubt that the control circuit exerts an influence on the relative intensities of the peaks of the resonances. This is borne out by the differences between the two series of resonance curves shown. Sometimes series 1 would be produced and at other times series 2, when switching on the multivibrator, no alterations whatever being made to the circuits.

It appears that the control voltage from the tuning fork can operate for two-phase relations between its sine wave of voltage and the discontinuous wave of the multivibrator. This is to be expected, since the multivibrator operations are similar in the two symmetrical halves of it and are separated in phase by 180 electrical degrees.

Looked at in this way the control may be considered as exerted sometimes on one half of the multivibrator and sometimes on the other half. No more than two series of harmonics could be obtained by switching on and off when everything else was kept constant. It is probable that a more symmetrical control would have been obtained by using a control frequency of 2,000 cycles per second, but this idea was not pursued owing to difficulties in constructing a highly accurate and permanent maintained tuning fork or other electro-dynamical system at such a frequency. The explanation of the variations in intensity almost certainly lies in the nature of the impact given to the system by the multivibrator. No records of the current in the anode circuit of fig. 3 were taken, but in fig. 5 is reproduced a curve of current in coil L1 in the grid circuit of

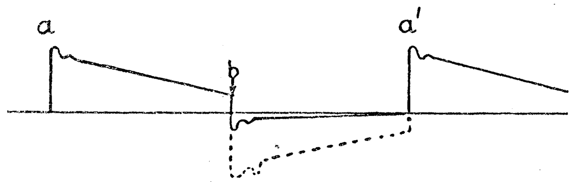

FIG. 5.—Current in coil L1 of fig. 3.

the multivibrator. The oscillatory portions at the discontinuities of the curve are due to insufficient damping of the shunted string of the galvanometer.

The same effect was observed when a simple make-and-break circuit replaced the multivibrator.

It will be seen from fig. 5 that a negative impulse is given to the selector circuit by the sudden smaller change in grid current at " b." If the positive and negative impacts alternate at exactly equal intervals of time, the odd harmonics will be greater in intensity than the even ones throughout the whole spectrum of harmonics ; if, however, " b " does not come exactly midway between " a " and " a'," then a distribution of intensities such as is shown in fig. 3 can be exactly accounted for. Strong support was given to this reasoning by applying a polarising negative potential to the grid of one valve of the multivibrator. This caused a great shortening in the active time of flow of the grid current. The corresponding belt of harmonics—not reproduced—showed abnormal augmentation of intensity of certain harmonics, say, n, $n + 2$, $n + 4$, and corresponding reduction of intensity of those of order $n + 1$, $n + 3$, etc. Considerable experimentation was made to try and get approximate equality of successive harmonics over a long range, but no successful way of doing this was found. A condition similar to that given by harmonics 25 to 30 in fig. 4, series 2, could always be obtained by adjustment of the multivibrator condensers for any desired region, but on either side of the belt so obtained the alternately weak and strong characteristic always showed itself. This characteristic of the harmonics, whilst, of course, not desirable, does not in practice introduce any uncertainty into the operation of the apparatus. No measurable change in the setting of the condenser corresponding to resonance at any selected harmonic was observed.

In further pursuance of the resonance curves given in fig. 4, some of these have been plotted on very much more open scales of condenser reading and current. Five adjacent harmonics are plotted in fig. 6. The full heavy line curve is the experimental one on harmonics 25 to 29 as measured by the thermogalvanometer. The three dotted resonance

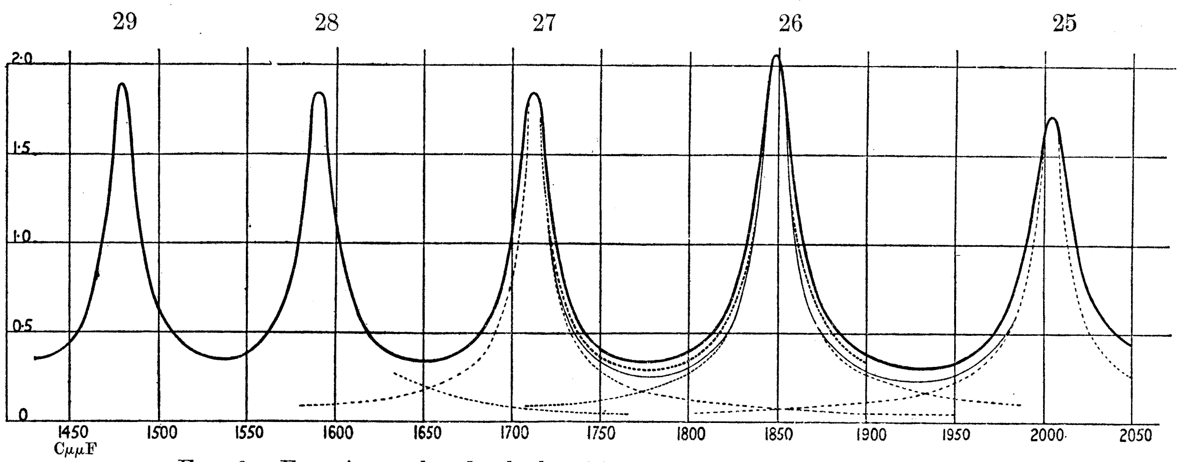

Fig. 6.—Experimental and calculated induced currents in selector circuit.

curves are theoretical curves, each giving the variation of current with variation of capacity, which would result for the case of a sine wave of impressed electromotive force of the same frequency as the selected harmonic, and of such value as to give a maximum current at resonance equal to the maximum current actually induced and measured for that harmonic, in the oscillatory circuit under the influence of the induced impulsive electromotive force of the multivibrator. These dotted curves have been calculated, using the well-known equation of the resonance curve for variation of capacity. The most convenient form of equation for this purpose is the exact one

$$I_1 = I_{\text{res.}} \frac{\delta \times C_1}{\sqrt{(\delta \times C_1)^2 + (\pi C_2)^2}}, \quad \dots \dots \quad (1)$$

in which δ = log dec. of the oscillatory circuit per complete period ;

C_1 = any capacity value whatever ;

C_2 = difference between C_1 and the resonance value of capacity without regard to sign.

The value of δ for the oscillatory circuit was determined both by direct delineation of a resonance curve, using a sine wave source set to the appropriate frequency, and also by calculation from the previously measured effective resistance of the coil and thermogalvanometer heater. The results agreed to within 1 or 2 per cent. The actual value of δ was 0·011. Three complete resonance curves are shown for frequencies of 25, 26 and 27 kilocycles per second.

From these curves the thin full-line curve has been constructed, by calculation of the value of

$$\sqrt{I_{25}{}^2 + I_{26}{}^2 + I_{27}{}^2 + \text{etc.}},$$

in which I_{25} represents the current at 25 kilocycles per second for any selected capacity, and I_{26}, I_{27}, etc., represent the similar quantities for these harmonics at the same capacity.

It will be seen that the resulting curve, although of exactly the same general form as the experimental one, does not exactly fit it, but represents a better case than is found experimentally, since the peaks are sharper and the troughs deeper. The reason for the discrepancy is that, in order to obtain sufficient deflection on the thermogalvanometer, it was necessary to couple the inducing coil L rather more closely than was desirable. The effect of the close coupling is, of course, that the component of current in the coil L at the frequency of any harmonic selected is reduced, as the selector circuit is brought into resonance at that frequency owing to the relatively large current induced in L at resonance. In order to obtain a more accurate idea of the extent of this effect, the mutual inductance between the inducing and selector coils was measured and an estimate of the reduction in current in the selector circuit due to reaction was calculated.

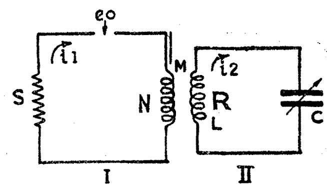

FIG. 7.—Simple circuit representing the valve and selector circuit of fig. 3.

The circuits involved may be considered to a sufficient approximation to be as in fig. 7. We may assume the inducing coil and anode circuit to be a closed circuit in which an electromotive force e_0 of value unaffected by circuit II operates.

Let i_1 and i_2 be the instantaneous values of current in the two circuits respectively. The letters indicating the constants of the circuits are self-explanatory. Putting $\alpha = j\omega$ we then have

$$i_1 (S + N\alpha) + i_2 M\alpha = e_0, \quad . \quad . \quad . \quad . \quad (2)$$

$$i_2 \left(R + L\alpha + \frac{1}{C\alpha}\right) + i_1 M\alpha = e_0, \quad . \quad . \quad . \quad . \quad (3)$$

substituting in (2) for i_2 its value given by (3) we have

$$i_1 \left[\frac{(S + N\alpha)(RC\alpha + q) + M^2\omega^2 C\alpha}{RC\alpha + q}\right] = 0, \quad . \quad . \quad . \quad . \quad (4)$$

in which $q = 1 - LC\omega^2$.

By the usual process this gives the approximate result

$$\frac{E_0^2}{I_1^2} \doteqdot S^2 + N^2\omega^2 + 2M^2C\omega^4 \left[\frac{SRC - Nq}{R^2C^2\omega^2 + q^2}\right]. \quad . \quad . \quad . \quad . \quad (5)$$

This gives approximately

$$\frac{I_1}{I_1'} \doteqdot 1 - \frac{M^2C\omega^4}{S^2}\left[\frac{SRC - Nq}{R^2C^2\omega^2 + q^2}\right] \quad . \quad . \quad . \quad . \quad (6)$$

where I'_1 represents I_1 for the case where $M = 0$.

In this expression $N^2\omega^2$ has been neglected as it is small in comparison with S^2.

The values of the quantities involved are, to a sufficient accuracy :—M (measured) = $1 \cdot 3 \times 10^{-3}$ henry.

$C = 2 \times 10^{-9}$ farad. $\omega = 16 \times 10^4$. $R = 10$ ohms. $S = 5 \times 10^4$ ohms. $N = 2 \times 10^{-2}$ henry.

These values have been inserted in equation 5 and a series of corrected values of I_2 calculated for various values of " q." (It is clear, of course, that I_2 will be reduced just in proportion to the reduction in I_1.) For the case where $q = 0$, *i.e.*, at the tip of the resonance curve, I_1 is less than I_1' by about 9 per cent.

The corrected curve for the experimental case (thick line) is drawn as a thick dotted line, again reducing the results so that the peak has the same value. It is now seen that the trough between the peaks comes much nearer the theoretical one, the difference between the curves is little greater than the inaccuracy of observation, and the whole peak is sharpened up to almost the exact theoretical shape. The desirability of keeping the coupling weak between multivibrator and selector is well brought out by these experiments. The value for the case taken was approximately $0 \cdot 065$, but in the actual wave-meter a very much smaller value has been used and the log. dec. of the selector circuit has been made as small as possible by careful design of the coils—to give as small time constant as possible within reasonable limits of diameter and weight of copper.*

The graphical treatment given above for the effect of an electrical impact on an oscillatory circuit, though very elementary, is sufficient to indicate that under the influence of the successive electrical blows given by the multivibrator the oscillatory circuit behaves substantially exactly as though sine-wave electromotive forces acted upon it, of frequencies and amplitudes having those values which would give the peak values obtained experimentally.

Prof. ABRAHAM has worked out mathematically the case of a system of multivibrator and selector circuit coupled to an amplifier, and has given expressions for the instantaneous quantity " q " on the condenser of the selector circuit as a function of the various electrical constants involved. Omitting the determination of the r.m.s. value of the current in the oscillatory circuit, he has obtained the expression for the electromotive force induced in the amplifier input circuit.

Starting from the expression for the quantity " q " at any instant in the selector circuit, we can easily find the root mean square current in this circuit. This should enable one to obtain the curve given in fig. 6.

The expression for " q " as defined above, and as given by Abraham, is

$$q_2 = \mathrm{B}e^{-at} \cos (\omega t + \psi) \quad \quad (7)$$

in which

$$\mathrm{B}^2 = \frac{\mathrm{Q}^2_0 (\omega^2 + \alpha^2)}{\omega^2 (1 - 2e^{-\alpha\theta} \cos \omega\theta + e^{-2\alpha\theta})} \quad \quad (8)$$

* Since making these observations it has been found that a value of 30,000 for S more nearly represents the truth. If this value had been used the corrected curve would have almost precisely fitted the experimental curve of fig. 6.

and

$$\tan(\psi - \phi) = \frac{e^{-\alpha\theta} \sin \omega\theta}{1 - e^{-\alpha\theta} \cos \omega\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

also

$$Q_0 = \frac{M}{L_2} q_0 \quad \text{and} \quad \tan \phi = \frac{\alpha}{\omega}$$

where q_0 = instantaneous quantity delivered in the operating circuit of the multi-vibrator,

θ = time interval between successive blows,

α = $R_2/2L_2$ (R_2 and L_2 resistance and inductance of the selector circuit),

and M = mutual inductance between the multivibrator coil and the selector circuit coil.

Starting from equation (7), we find for the instantaneous current in the selector circuit

$$i_2 = \frac{dq_2}{dt} = - Be^{-\alpha t} [\alpha \cos(\omega t + \psi) + \omega \sin(\omega t + \psi)] \quad . \quad . \quad . \quad . \quad (10)$$

which reduces to

$$i_2 = - Be^{-\alpha t} \sqrt{\alpha^2 + \omega^2} \cos(\omega t + \lambda) \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

in which $\lambda = \psi - \gamma$ and $\tan \gamma = \omega/\alpha$.

Calling I_2 the root mean square current, we have

$$I_2^2 = \frac{1}{\theta} \int_0^\theta i_2^2 \, dt$$

from which

$$I_2^2 = \frac{B^2}{\theta} (\omega^2 + \alpha^2) \int_0^\theta e^{-2\alpha t} \cos^2(\omega t + \lambda) \, dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This may be written

$$I_2^2 = F^2 \int_0^\theta e^{-2\alpha t} \left[\tfrac{1}{2} + \frac{\cos 2(\omega t + \lambda)}{2} \right] dt, \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

giving

$$I_2^2 = \frac{F^2}{4} \left\{ -\frac{1}{\alpha} \left[e^{-2\alpha t} \right]_0^\theta - \frac{1}{\sqrt{\omega^2 + \alpha^2}} \left[e^{-2\alpha t} \cos 2(\omega t + \lambda + \xi/2) \right]_0^\theta \right\}, \quad . \quad . \quad (14)$$

in which $\xi = \tan^{-1} \omega/\alpha$, whence

$$I_2^2 = \frac{B^2}{4\theta} (\omega^2 + \alpha^2) \left\{ \frac{1}{\alpha} (1 - e^{-2\alpha\theta}) - \frac{1}{\sqrt{\alpha^2 + \omega^2}} [e^{-2\alpha\theta} \cos 2(\omega\theta + \lambda + \xi/2) - \cos 2(\lambda + \xi/2)] \right\}. \quad . \quad . \quad . \quad . \quad (15)$$

In the present case $\alpha = \dfrac{R_2}{2L_2} = n\delta \doteqdot 275$ for the 25th harmonic; but $\omega \doteqdot 2\pi n \doteqdot 15 \times 10^4$, so that $\dfrac{1}{\sqrt{\omega^2 + \alpha^2}}$ is negligible compared to $\dfrac{1}{\alpha}$.

Again, for values of α and θ of 275 and 1×10^{-3} respectively $(1 - e^{-2\alpha\theta})$ becomes 0·4231, and the cosine expression in the second bracket cannot be greater than 1·42 so that the whole second term is always less than about 0·5 per cent. of the term $\frac{1}{\alpha}(1 - e^{-2\alpha\theta})$ and may therefore be omitted. Inserting the value for B and neglecting α^2 in comparison with ω^2 we get finally

$$I_2^{\,2} = \frac{Q_0^{\,2}}{4\alpha\theta} \cdot \frac{\omega^2(1 - e^{-2\alpha\theta})}{1 + e^{-2\alpha\theta} - 2e^{-\alpha\theta}\cos\omega\theta} . \qquad (16)$$

An examination of this expression shows immediately that $I_2^{\,2}$ cycles up and down through maximum and minimum values as $\cos\omega\theta$ passes through its limiting values of $+1$ and -1, corresponding to values of ω equal to $\pi k/\theta$, where k is any integral number whatever. The resonance peaks correspond to values of ω of $2\pi k/\theta$ giving angles, 0, 2π, 4π, etc. The troughs correspond to values of ω midway between the peaks, so that $\cos\omega\theta = -1$.

The agreement of the theory with experiment is seen by calculating, as an example, the ratio of maximum to minimum current for the 25th harmonic and following trough.

The values of the quantities involved are

$$\alpha = R/2L \doteqdot 275 \qquad \theta = 1 \times 10^{-3}\ \text{sec.}$$

$$\left(\frac{I_{\text{max.}}}{I_{\text{min.}}}\right)^2 = \frac{1 + e^{-2\alpha\theta} + 2e^{-\alpha\theta}}{1 + e^{-2\alpha\theta} - 2e^{-\alpha\theta}} = \frac{1\cdot577 + 1\cdot519}{1\cdot577 - 1\cdot519} = 53\cdot4,$$

whence $I_{\text{max.}}/I_{\text{min.}} = 7\cdot3$.

The value found experimentally is 7·0, taking the ratio between the mean of the peaks of the 25th and 26th harmonics and the intervening trough, and correcting for the mutual effect.

Another case was worked out for the 40th harmonic, at which frequency the effective resistance R_2 of the selector circuit is about 14 ohms, giving for "α" the approximate value of 350. The calculated ratio of maximum current to minimum current is 5·8, and the observed ratio is about 5, thus, again, giving quite satisfactory agreement, bearing in mind that actual experiment shows the undulation in the peaks as given in fig. 4.

If we examine the formula (16) it is seen that the ratio of maximum $I_2^{\,2}$ to the adjacent minimum $I_2^{\,2}$ is equal to

$$\frac{1 + e^{-2\alpha\theta} + 2e^{-\alpha\theta}}{1 + e^{-2\alpha\theta} - 2e^{-\alpha\theta}} \qquad (17)$$

and that this ratio is only dependent on α and θ. Now $\alpha = R/2L$ is equal to $n\delta$ where δ = log. dec. of the selector circuit per complete period. But $n = k/\theta$, where k is any integer and n = frequency.

Hence, for the reinforcement ratio of a harmonic as given by (17) we may write

$$\left(\frac{I_{\text{max.}}}{I_{\text{min.}}}\right)^2_k \doteqdot \frac{1 + e^{-2k\delta} + 2e^{-k\delta}}{1 + e^{-2k\delta} - 2e^{-k\delta}} \qquad (18)$$

This shows that both δ and k should be as small as possible. The indication is, too, that as k becomes larger at the higher order harmonics it is desirable for δ to become smaller.

Since, however, there is difficulty in making " δ " small enough in any case, it is probably the best compromise to design the coils to have minimum decrement somewhere towards the upper range of frequency at which each will be used.

With regard to the factor " k," from the point of view of reinforcement as defined by equation (18), there is no doubt that it is undesirable to make use of harmonics of too high order. On the other hand, when " k " is, say, only 10, the successive harmonics are 10 per cent. apart in frequency, thus leaving rather a large gap for purposes of interpolation. As will appear later when using a fundamental of 1,000 cycles per second, there are available sub-harmonics in between the main integral harmonics. With a higher fundamental frequency slightly modulated by a lower harmonic frequency, as occurs in the complete wave-meter, one can also obtain sub-harmonics for a considerable distance on each side of the main harmonic and so bridge a large portion of the intervening region between two harmonics.

3. Comparison Experiments between Frequencies given by the Multivibrator and those obtained by Calculation from Inductance and Capacity.

It was felt very desirable to make a comparison of the radio-frequencies given by the multivibrator with those obtained by calculation from the inductance and capacity of the resonant selector circuit. The effective inductance and effective capacity of the circuit must, of course, be independently measured in terms of some fundamental standard, and they must be expressed in the same system of units. The circular check thus obtained on all the measurements made becomes of great value in strengthening the general position as regards measurement of frequency, inductance and capacity.

It will be convenient to describe first, very briefly, the measurements on the standard coils and variable condenser before proceeding to the radio frequency comparisons.

The foundation of the inductance and capacity measurements at the National Physical Laboratory is a standard of mutual inductance calculable from dimensions. This was designed and described by A. Campbell (10). A second similar one was constructed and comparisons were made (11) by the author with the original standard. The results showed that the true value of mutual inductance of the standard in c.g.s. units is probably within one part in a hundred thousand of the value obtained by calculation from the measured dimensions.

The inductance coils were measured with low-frequency current on a mutual inductance bridge with equal arms by the help of a secondary sub-divided standard mutual inductometer carefully calibrated from the primary standard. These measurements presented no special features; care was taken at every point by reversals of source, galvanometer, ratio arms, etc., to eliminate all cross effects. It is considered that the values of self-inductance of the coils so measured are accurate to five parts in a hundred thousand.

The more difficult calibration of the standard variable air condensers was made by means of a Carey-Foster bridge determining capacity in terms of mutual inductance and resistance. The addition of a Wagner earthing device, first used on this bridge by the author, renders the effects of earth capacities of the various parts of the bridge of negligible account. A diagram of this bridge with the earthing device is given in fig. 8. It is seen that the bridge measures capacity in terms of quantities whose dimensions are [M]/[R]². In order to be consistent, M and R must be expressed in the same system of units. M, and hence L, is known in c.g.s. units, and since C is also required in the same system it is necessary to know R in these units. The resistance coils used were measured in the ordinary way in international ohms against the Laboratory standards and were corrected to absolute c.g.s. ohms, using the ratio of units determined by F. E. SMITH (12). The measurements of capacity are not of such high order as those of inductance, but it is believed that the absolute accuracy of the calibrations at various readings on the condenser is to a few parts in ten thousand. As will be seen later on, only changes of capacity of the condenser are involved in the comparison with the radio-frequency tests, so that uncertainties regarding capacity of the coil and leads to the condenser and of these to earth are thrown on to the effective self-capacity of the coil and do not vitiate the accuracy at all. The mean of differences in capacity between readings on the condenser is probably accurate to one or two parts in ten thousand for the average of twenty observations at different parts of the scale.

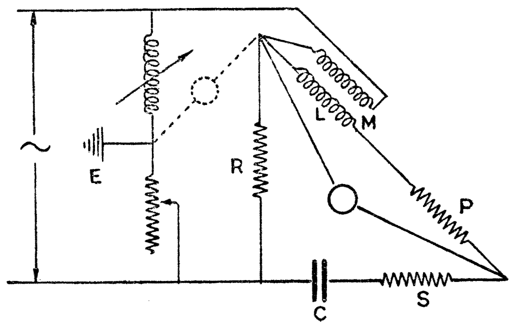

FIG. 8.—Carey Foster bridge with Wagner earth.

Passing on to the radio-frequency tests, the arrangements for these were as given in fig. 9.

Four distinct items of apparatus were used in these experiments, as follows :—

I. The multivibrator M (shown purely diagrammatically) with its controlling tuning-fork F.

The tuning-fork had its own battery for the filament of its valve, so as to be as constant as possible. The multivibrator coil L1 was connected by leads about a metre long, so that circuit II might be well removed from metal parts and coils in circuit I. The coupling between I and II was very loose.

II. This circuit was the standard circuit under observation and was reduced as nearly as possible to an elemental form. The capacity part consisted of the standard calibrated variable air condenser. The inductance part was one of the standard inductance coils which had also been measured at a low frequency. These coils were specially suited to the purpose in view, being very permanent, of highly stranded wire, of small decrement and of small self-capacity.

III. This was a six-valve transformer coupled amplifier for the purpose of detecting the resonances in circuit II. It was very loosely coupled to the standard circuit by the coil L3. The position of this coil was adjusted so as to have no mutual inductance to coil L1.

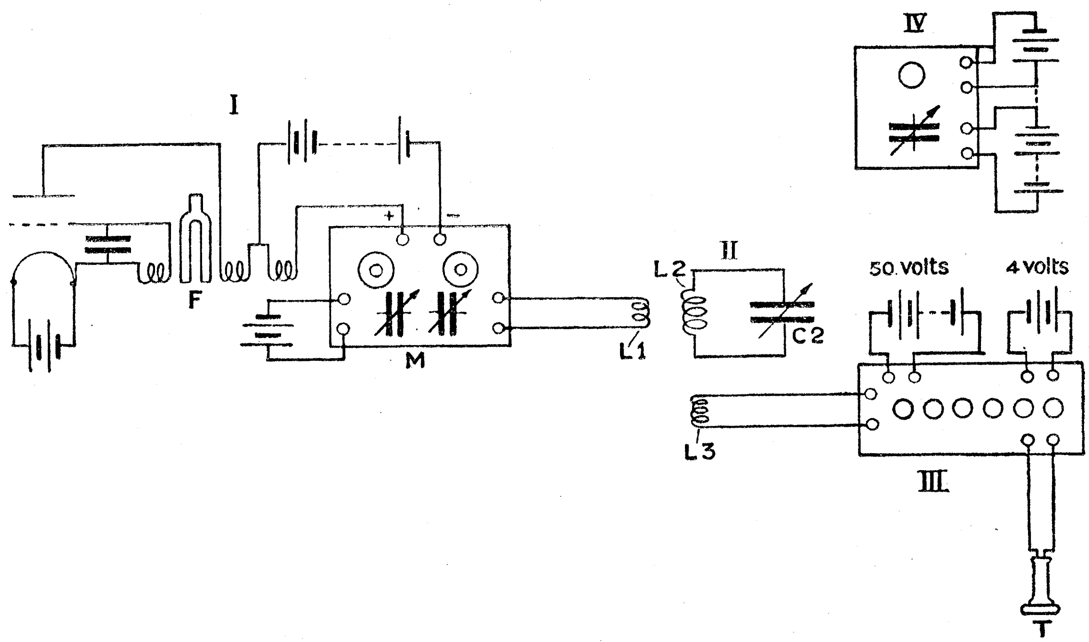

FIG. 9.—Arrangement of apparatus for determination of resonant settings of standard selector circuit.

Unit IV was an ordinary heterodyne set with its own batteries, and was provided with fine setting on its variable condenser in order that the interfering beat tone with the chosen harmonic might be adjusted to any desired acoustic frequency.

Observation of the Resonances.

The setting of the condenser to resonance at the chosen harmonics was carried out in four different ways altogether, as follows :—

Method 1.—The heterodyne unit IV was not in use and the standard condenser was set until maximum loudness of the 1000 ∽ per second note in the amplifier telephone occurred. This note results, of course, from the rectification, by the amplifier, of the modulated wave in the standard oscillatory circuit.

ABRAHAM has shown in his theory of the multivibrator that the setting of the condenser for maximum loudness of the rectified modulated current in the amplifier telephone is not quite the same as that for resonance by maximum loudness of the interference tone with a separate heterodyne source. The correction term is given by $C_1 = C_0 (1 - 5/4\ \delta^2/\pi^2)$ where δ = log. dec. of the selecting circuit.

In the present case " δ " was of the order 0·01, so that the correction term is only of the order one part in a hundred thousand, and is therefore entirely negligible.

Method 2.—This was a refinement of method 1, in that a Duddell vibration galvanometer tuned to a frequency of 1000 cycles per second replaces the telephone. This means of observing resonance was, of course, very much more accurate than the foregoing. Owing to the very slight uncertainty as to whether the maximum intensity of the 1,000 cycle per second note heard in the amplifier telephone really represented maximum oscillatory current in the standard circuit, it was decided to use methods 3 and 4, requiring the heterodyne.

Method 3.—In this method the intensity of the beat tone between the chosen harmonic and the adjusted heterodyne source is caused to have a maximum intensity as judged by loudness in the amplifier telephone.

Method 4.—This is a refinement of method 3, in that a Duddell vibration galvanometer tuned to a frequency of about 800 cycles per second was used instead of the telephone, and the heterodyne frequency was accurately set so that the difference tone was at the resonant frequency of the galvanometer. This adjustment had, of course, to be made with great precision. When made, the standard variable condenser of the selector circuit II was set to give maximum deflection on the vibration galvanometer. This last method was the most accurate of all, but was extremely laborious, occupying many hours for a series of 60 harmonics.

Determination of the Order of the Harmonics.

This was really unnecessary in these experiments because the effective values of inductance and capacity were known to an accuracy which left no uncertainty as great as 0·5 per cent. in the value of the frequency of resonance. Since the range of harmonics was over the region 20 to 100, and since they were all observed and carefully counted upwards from, say, the 20th, there was not the slightest doubt regarding the order number of the harmonic.

In order to render this fundamental point doubly certain, the method given by ABRAHAM was also used and found to give absolutely unmistakable location to the harmonic. The method is very simple: a harmonic, say the nth, is selected and the heterodyne is set to give a fairly low frequency beat tone of about 400 per second. The succeeding harmonics are then selected one by one and counted, care being taken not to miss one. When the 2 nth harmonic is reached, an interference beat tone of 800 per second will be heard against the 2nd harmonic of the heterodyne. The number of intervals counted between "n" and "$2n$" is, of course, equal to "n" the order number of the harmonic first selected. Some little skill is required in order to select the correct interference tones at each end of the region counted. All the notes such as -2400, -1400, -400, $+600$, $+1600$, can be heard corresponding to beats between the heterodyne and the successive harmonics $n-2$, $n-1$, n, $n+1$ and $n+2$. Similarly at the other end of the scale the notes -1800, -800, $+200$, $+1200$ can be heard corresponding to the selected harmonics $2n-1$, $2n$ and $2n+1$.

The observations of condenser readings corresponding to resonance for a series of selected harmonics have been dealt with as follows. If resonant capacity is plotted as abscissa and $1/n^2$ as ordinate (where n = frequency), a straight line is obtained cutting the " X " axis at a small negative value of C. This intercept represents the self-capacity of the coil and includes any stray capacities outside the screened condenser. The slope of the line is equal to the effective pure inductance of the coil. If the coil is of finely stranded wire the effective pure inductance so determined should equal the inductance measured at low frequency in terms of the mutual standard. It is not possible to determine the slope with sufficient accuracy by graphical methods, but by the method of least squares quite high accuracy may be obtained.

The formulas for effective pure inductance and effective self-capacity have been developed by HULBERT and BREIT (13). They are, in terms of frequency,

$$L = \frac{10^{18}}{4\pi^2} \frac{N\Sigma\left(\frac{1}{n}\right)^4 - \left[\Sigma\left(\frac{1}{n}\right)^2\right]^2}{N\Sigma C_1 \frac{1}{n_1^2} - (\Sigma C_1)\left[\Sigma\left(\frac{1}{n_1}\right)^2\right]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$C_0 = \frac{(\Sigma C_1)\left[\Sigma\left(\frac{1}{n_1}\right)^4\right] - \left[\Sigma\left(\frac{1}{n_1}\right)^2\right]\left[\Sigma C_1\left(\frac{1}{n_1}\right)^2\right]}{\left(\Sigma\frac{1}{n_1^2}\right)^2 - N\Sigma\frac{1}{n_1^4}}.$$

These equationsassume that the percentage accuracy of condenser reading is proportional to the capacity. This method is very laborious to apply owing to the large amount of arithmetic involved. It has been used in a few cases, but in other cases a method has been used in which an assumed value of L is taken and theoretical capacities calculated corresponding to the frequencies of the harmonics. The difference between these values and the observed value gives a capacity which includes the self-capacity of the coil, etc., and the error of observation. The mean value of these differences is next taken and a second difference representing error of observation found. The sum of these second differences is finally taken. By choosing other values of L and repeating the

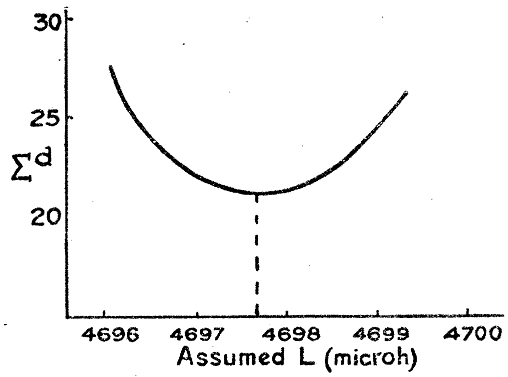

FIG. 10.—Deduction of effective inductance of an oscillatory circuit.

process a curve can be obtained between assumed L and (Σ 2nd differences). The value of L corresponding to the minimum Σ difference gives the best value.

A typical series of such values is shown by the curve in fig. 10.

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A collection of results is given in Table III which also gives the value of L obtained by comparison against the standard mutual inductance.

TABLE III.

Coil.	Method of observing Resonance.	Effective Inductance.	Mean Value.	Effective Inductance by Mutual Standard.
L 307	1 3 4	7843·4 7843·3 7844·8	$7843._8$	7844
Ad. 7	3 3 3	4698·0 4697·7 4698·0	$4697._9$	$4698._2$
L 450	—	20014	—	20017

The observations for the coil No. L 450 were those corresponding to the experiments described in section 2, where the relative amplitudes of the harmonics are plotted in fig. 4, and were taken by direct observation of the current peaks as given by the thermo-galvanometer.

In order to check whether a wrong observation had been made, the value of L deduced by the least squares was used to calculate the frequencies by the ordinary formula, using observed values of capacity as written down, and adding to each, of course, the value of self-capacity also determined. In order to give an idea of the accuracy obtained, the observations and calculated values of frequency are given for a complete set of harmonics in Table IV.

TABLE IV.

Harmonic	C. rdg. $\mu\mu$F.	Calculated Frequency.	True Frequency.	Difference, Parts in 1000.
16	4 923	16 001	16 000	+ 0·06
17	4 358	17 001	17 000	+ 0·06
18	3 885	18 001	18 000	+ 0·06
19	$3\ 483._5$	19 002	19 000	+ 0·10
20	3 140	20 007	20 000	+ 0·35
21	2 849	20 996	21 000	− 0·20
22	2 591	22 008	22 000	+ 0·37
23	2 371	22 995	23 000	− 0·23
24	$2\ 178._5$	23 983	24 000	− 0·73
25	2 006	24 978	25 000	− 0·89
26	1 850	25 995	26 000	− 0·20
27	1 711	27 015	27 000	+ 0·55
28	1 590	28 009	28 000	+ 0·31
29	1 480	29 015	29 000	+ 0·52
30	1 381	30 030	30 000	+ 1·00

Table IV (continued).

Harmonic	C. rdg. $\mu\mu$F.	Calculated Frequency.	True Frequency.	Difference, Parts in 1000.
31	1 290.$_5$	31 040	31 000	+ 1·30
32	1 211	32 017	32 000	+ 0·53
33	1 140	32 978	33 000	− 0·67
34	1 070	34 016	34 000	+ 0·48
35	1 010	34 988	35 000	− 0·29
36	951	36 030	36 000	+ 0·82
37	900	37 010	37 000	+ 0·27
38	850.$_5$	38 042	38 000	+ 1·10
39	809	38 977	39 000	− 0·59
40	765	40 047	40 000	+ 1·17

4. Crucial Test of the Multivibrator Principle.

In order to place beyond all doubt any question as to the validity of the principle of building-up frequency by harmonic ratios, the following test was carried out:—

Two complete multivibrator assemblages were set up; these will be designated A and B respectively.

Assemblage A consisted of a complete compound multivibrator* having a primary control tuning-fork of a frequency of approximately 1,000 vibrations per second. The 20th harmonic was selected from the low-frequency multivibrator and controlled a high-frequency multivibrator of fundamental of 20 kilocycles per second. From this, any harmonic, say the 12th, could be selected on a selector circuit. A tuned amplifier was loosely coupled to the selector circuit.

Assemblage B was similar to A, but the control tuning-fork had a frequency of approximately 969 vibrations per second. The 19th harmonic of the low-frequency multivibrator was selected and used as control for the high-frequency multivibrator. The fundamental frequency was therefore approximately 18·4 kilocycles per second. A selector circuit enabled one to select any desired harmonic from this high-frequency multivibrator. In both multivibrator assemblages the order of harmonic was determined absolutely, *i.e.*, without assuming any frequencies to be known.

A local source of oscillations was also provided and was of a kind that kept very constant in frequency when adjusted. The capacity part of its oscillatory circuit was provided with an open-scale standard variable condenser in parallel with the main capacity. Calling the capacity reading on this generator condenser C2, the following procedure was carried out. A region of frequency was found in which a selected harmonic of A was close to one of B.

* For a complete understanding of this arrangement it is necessary to refer to Sections 5 and 6 and to Fig. 11.

The frequency of the source was adjusted to exact synchronism with a main harmonic of A and with a number of subsidiary harmonics on either side. (These can be given sufficient prominence by increasing the control of the high-frequency multivibrator by the low one.) The readings on C2 corresponding to these sub-harmonics were noted.

The same procedure was now carried out using B and so obtaining an interlacing series of harmonics. The readings for multivibrator A were then repeated immediately after those taken with B, so as to check any drift in the local source.

A note may be interposed here regarding the observation of harmonics and subsidiary harmonics. The frequency of the fork A may be assumed equal to 1000·00 cycles per second $= n_A$. The frequency of the main harmonic used on the high-frequency multivibrator, for example, the 12th, then becomes $20 \times 12 \times 1000 \cdot 00 =$ 240·00 kilocycles per second. The subsidiary harmonics which can be observed are very many; for example, not only can those corresponding to frequencies of $240{,}000 \pm k \times n_A$ be observed, where $k = 1, 2, 3, 4$, etc., but secondary subsidiary intermediate harmonics may also be utilised such as $240{,}000 \pm k \,.\, n_A \pm r \,.\, n_A$, where r is a fraction, such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. Thus one can set the source exactly to a frequency such as 241250 without the aid of any auxiliary tuning-forks. The possibility of obtaining fiducial points so close together is very valuable because it removes practically all responsibility of accuracy in the generator condenser C2 when interpolating. Some skill is naturally necessary in interpreting the beats heard, to ensure which sub-harmonic and secondary sub-harmonic is producing the slow beat heard.

Returning now to the method of dealing with the readings on C2, the following procedure was adopted:—

The frequency of the controlling tuning-fork on multivibrator A was assumed equal to 1000·00 vibrations per second. The main harmonic used on the high-frequency multivibrator was the 12th, so that the frequency of this point becomes $20 \times 12 \times 1000 \cdot 00 = 240 \cdot 00$ kilocycles. The other sub-harmonics observed had nominal frequencies, as given in Table V below.

TABLE V.

Multivibrator A.		Multivibrator B.	
Nominal Frequency. Kilocycles per Second.	Mean C2 Reading. $\mu\mu$F	Harmonic Observed.	C_2 Reading. $\mu\mu$F.
238·00	1,579	—	—
238·50	1,397	245·50	1,582
239·00	1,219	246·00	1,407
239·50	1,039	246·50	1,234
240·00	863	247·00	1,060
240·50	687	247·50	$890._5$
241·00	512	248·00	720
241·50	336	248·50	550
242·00	166	249·00	380

Column 4 gives the condenser readings C2 for the harmonics observed on multivibrator B. In this case the 13th harmonic of the high-frequency multivibrator was used, and this multivibrator was controlled by the 19th harmonic of the low-frequency multivibrator B, having a fundamental frequency of approximately 969 cycles per second.

These harmonics therefore have frequencies of $(13 \times 19)\, n_B \pm k n_B$, as shown in the table ($k = 0{\cdot}5$, $1{\cdot}0$, $1{\cdot}5$, etc.). The frequencies of these harmonics were obtained by interpolation in terms of multivibrator A.

If now we divide each interpolated frequency by the harmonic number of B, we should obtain in every case the fundamental frequency of tuning-fork B. The results of these calculations are given in Table VI.

TABLE VI.

Multivibrator B.

Observed Harmonic.	Interpolated Frequency. Kilocycles per Second.	Deduced Fork Frequency. Cycles per Second.
245·50	$237{\cdot}98_4$	969·38
246·00	$238{\cdot}47_3$	969·40
246·50	$238{\cdot}95_8$	969·40
247·00	$239{\cdot}44_2$	969·40
247·50	$239{\cdot}92_1$	969·38
248·00	$240{\cdot}40_6$	969·38
248·50	$240{\cdot}90_2$	969·42
249·00	$241{\cdot}37_0$	969·36

Mean temperature, 18·60° C. Mean fork frequency = $969{\cdot}39_0$

Chronograph runs were taken on the two tuning-forks by the help of a specially constructed phonic wheel suitable for frequencies of the order of 1,000 per second. By this means the frequency of each fork was determined, to an accuracy of one part in a hundred thousand, in terms of the second given by the standard clock of the Laboratory. The values obtained for the forks were :—

By Chronograph .. Fork A at 20° C. = $999{\cdot}96_5$ cycles per second.
,, ,, .. ,, B at $18{\cdot}5_5$° C. = $969{\cdot}34_5$,, ,,

The mean value obtained for Fork B by the process of stepping up and down through the multivibrators was :—

Fork B at 18·6° C. = $969{\cdot}39_0$ in terms of Fork A of nominal value $1000{\cdot}00_0$. Correcting these results for the error of A and to a temperature of $18{\cdot}5_5$° C., we get :—

By multivibrator .. A, Fork B at $18{\cdot}5_5$° C. = $969{\cdot}36_0$ cycles per second.

This agreement may be considered very satisfactory and can be considered to remove every trace of uncertainty as to the validity of the harmonic principle of building-up frequency in view of the fact that four harmonic stages intervene in the calculation and measurement of fork B in terms of fork A.

5. Fundamental Principles Governing the Design of the Wave-meter.

The experiments and investigations just described gave a great deal of valuable experience and information for guidance in the design of the complete self-contained instrument. Before proceeding to details it is desirable to set down clearly the behaviour of the wave-meter as used to set the frequency of another source or to measure it. This may be summarised as follows :

(*a*) *For Frequencies up to* 120 *Kilocycles per Second.*

A low-frequency multivibrator unit produces a fundamental frequency of 1 kilocycle per second. The frequency of operation of this multivibrator is directly controlled by an electrically maintained tuning-fork, having a frequency which has been adjusted as closely as possible to 1,000 cycles per second. The impulse from this multivibrator acts by mutual inductance coupling of small amount on a selector circuit, which consists of a special variable condenser and either of three standard radio-frequency inductance coils. By means of directly marked scales any desired harmonic (from the 10th to the 120th) can be selected and reinforced.

The selected and reinforced harmonic is amplified by means of a tuned amplifier. This amplifier also receives a small amount of high-frequency current from any source of undamped oscillations.

This source may be a station or any other generator whose frequency is desired to be known or set to a particular value. We will assume that it is desired to set this source to some exact radio-frequency. The procedure consists in selecting that harmonic whose frequency lies near to that desired for the source. On adjusting the amplifier so that it is sensitive to this frequency, a beat tone will be heard between the selected harmonic and the source (assuming, of course, that the latter is sufficiently nearly adjusted already for this beat tone to be audible). The source can now be set until this beat tone becomes as slow as one in ten seconds, representing to all intents and purposes exact synchronism with the harmonic. Or it may be set to give any desired beat frequency from the selected harmonic representing a radio-frequency such as 50300. In such a case, a beat tone of — 300 cycles per second is heard against the 50th harmonic, when this one is selected, or a beat tone of + 700 per second may be obtained against the selected 51st harmonic, if so desired. For purposes of checking, beat tones of — 1300, — 2300 or + 1700, + 2700 cycles per second may also be observed against the appropriate harmonics. By this system any desired frequency can be obtained up to 120 kilocycles per second, with an accuracy of a few parts in a hundred thousand.

(*b*) *For Frequencies from* 100 *Kilocycles per Second to* 1,200 *Kilocycles per Second.*

For these higher frequencies a second multivibrator is provided. This apparatus is very similar to the low-frequency multivibrator apparatus, but produces a discontinuous wave of fundamental frequency of 20 kilocycles per second.

This apparatus is controlled in frequency by means of the reinforced and amplified 20th harmonic of the low-frequency multivibrator; for this purpose special circuits are incorporated into the apparatus. This high-frequency impulse operates on the selector circuit in the same manner as the low-frequency multivibrator. The same variable air condenser is used, but another set of three coils and scales is provided to suit the range 100 to 1,200 kilocycles per second, corresponding to harmonics 5 to 60. The scales have engraved lines corresponding to every harmonic from 5 to 60, but they are, of course, marked directly in kilocycles per second. The principle of controlling the two multivibrators in tandem from the tuning-fork is one of the most valuable features of the apparatus. The advantages are :—

(1) All the frequencies obtained are an integral number of kilocycles per second.

(2) The responsibility of the accuracy and permanence of the calibration is removed from all the electrical circuits except, in a very minor degree, those immediately associated with the tuning-fork.

(3) The operation of the apparatus is not affected by the presence of the observer, earth capacities, etc.

From the outline given above it will be seen that the complete equipment divides itself naturally into three separate parts :—

(*a*) The compound multivibrator unit.
(*b*) The selector unit.
(*c*) The detector and amplifier unit.

Short general descriptions of these units are now given.

6. Description of the Wave-meter.

(*a*) *The Multivibrator Unit.*

This unit consists of two multivibrators in cascade, together with the intermediate control circuit and the main controlling tuning-fork. The complete scheme of connections is given in fig. 11. For clearness of description this diagram is divided into four sections, lettered A, B, C and D, as follows :—

A. The tuning-fork is valve-maintained and has a fundamental frequency of 1,000 cycles per second. The grid and anode windings are provided with condensers of such values as to make the natural frequency of these circuits approximately equal to that of the fork. A winding L3 underneath the anode winding is provided for the purpose of introducing into the anode battery circuit of the multivibrator the small alternating control voltage which keeps this multivibrator synchronous with the fork.

The control thus provided is of such extent that synchronism is maintained over a range of variation in the multivibrator circuits, which in the absence of the control would cause a change in frequency of ± 4 per cent.

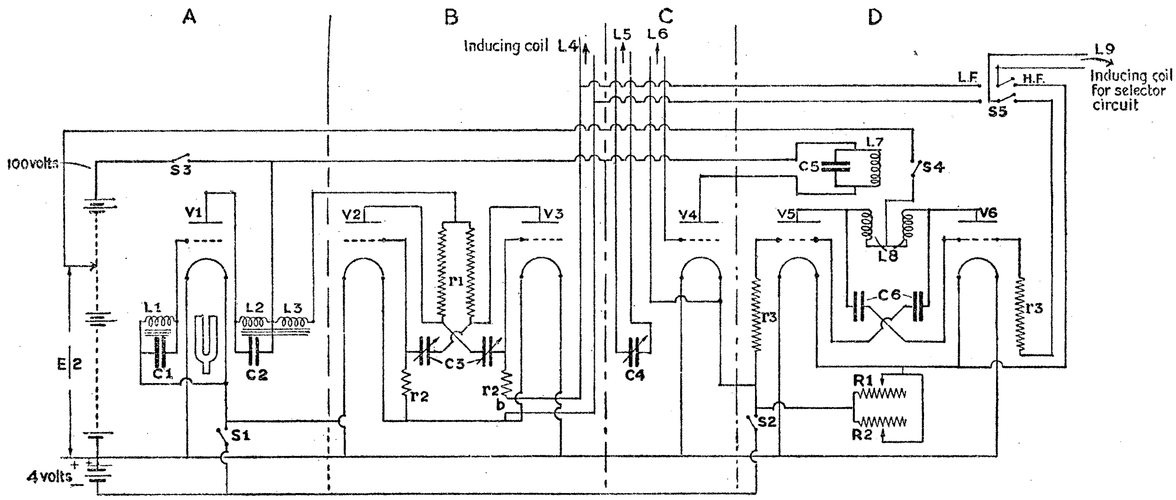

FIG. 11.—Arrangement and connections of tuning-fork controlled compound multivibrators.

The conditions governing steadiness of operation of the fork have formed the subject of a previous paper by the author (13). The present fork is the outcome of the conclusions reached in these experiments.

The fork is constructed of a nickel-iron alloy which has a very small temperature coefficient of elasticity. It has been cut from a wide flat bar of the material in such a manner that a large cross-section of metal is left at the roots of the prongs. The metal here extends beyond the outside surfaces of the prongs, so that they are supported at the root in such a manner that the nodal surfaces are of large area and are close together. The idea has been to reduce as much as possible the axial vibration of the support. Clamps of considerable weight (about 2 kgm.) hold the fork and its auxiliary parts in rigid relationship.

Experiments showed that a want of equality of stiffness of the prongs, or of their effective masses, manifested itself as a rotational oscillation of the base. The prongs were carefully balanced experimentally until this oscillatory vibration was reduced to a minimum. With the fork so constructed and balanced scarcely any vibration of any kind is conveyed outside the fork system. The change in frequency produced by clamping and unclamping the mounted fork at its attachment to the base of the complete apparatus was less than one part in a hundred thousand.

From the point of view of permanence the fork appears to be very satisfactory. During the eighteen months the fork has been under observation, its absolute frequency, measured by means of a special phonic wheel constructed for the purpose, has not shown variations from the mean frequency by as much as ± 2 parts in 100,000.

Turning again to fig. 13 we come to the section B.

B. This is the low-frequency multivibrator item consisting of anode resistances r_1

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of eureka wire of 50,000 ohms each, and condensers C3 cross-connecting grids and anodes of the valves V2, V3. These condensers each consist of a fixed mica condenser of 0·006 μF and a variable air condenser of 0·001 μF. maximum capacity, in parallel. The leak resistances r_2, r_2 are also of eureka wire on mica frames and are 75,000 ohms each. A break is made in the circuit of one of these at "*b*," at the filament end ; leads from this convey the impulse either to the controlling inducing coil L4 or to the terminals marked "Inducing coil for selector circuit" (top right corner of diagram) for this connection, the change-over switch S5 is to the left-hand side marked LF. Switch S1 is in the 4-volt filament circuit of valves V1, V2 and V3 and switch S3 is in the 100-volt anode battery circuit of these valves and valve V4.

When these switches are closed the tuning-fork and the low-frequency multivibrator are activated. By adjustment of C3 the frequency is brought within the range of synchronisation. This is not critical ; the region of holding is for a variation over half the scale of the variable part of C3.

The next portion, C, is the control arrangement for the high-frequency multivibrator.

C. Many different arrangements were tried in order to obtain satisfactory control of the high-frequency multivibrator D

The real difficulty was that strong control could not be obtained without, at the same time, producing in the high-frequency multivibrator a large modulation from the low-frequency multivibrator. (This effect will be discussed later. There are advantages and disadvantages attending the presence of sub-harmonics in the immediate neighbourhood with respect to frequency of a selected harmonic of multivibrator B.) The final method of control adopted was as follows :—The impulse from the low-frequency multivibrator B is diverted to L4 (at the top of the diagram in the middle) and operates upon a selector circuit consisting of an inductance coil on L5 and a condenser C4. Part of C4 is variable and enables this circuit to select the 19th, 20th or 21st harmonic of B. The 20th harmonic is the one normally selected. This reinforced harmonic induces into a coil L6 connected to the grid filament circuit of the valve V4. In the anode circuit of V4 is the tuned impedance circuit C5, L7 having fixed frequency of about 20 kilocycles per second. A current of a few milliamperes at a frequency of 20 kilocycles results in this circuit. The inductance coil L7 is loosely coupled to the two inductive anode coils L8 of the high-frequency multivibrator D. By this means an electro-motive force, at a frequency of 20 kilocycles per second and with small modulation at 1 kilocycle per second, is impressed on the anode circuits of D and will hold this multivibrator in step over a limited range. If the coupling between L7 and L8 is increased beyond a certain small value, the modulation at 1 kilocycle per second becomes troublesome, but the range of synchronisation is much increased. A compromise has been made whereby the range of synchronisation is the equivalent of a few parts in a thousand in frequency of multivibrator D.

With this amount of control the subsidiary harmonics are just noticeably present when harmonics above the 30th are selected. There is, however, no uncertainty whatever when setting an outside source to exact synchronisation with a main harmonic. The relationship between ease of control and freedom from modulation at the frequency of 1,000 cycles per second was made clearer by subsequent experiments on the high-frequency multivibrator.

D. This is the high-frequency multivibrator, having a fundamental frequency of 20 kilocycles per second. It consists of the following parts:—Anode inductive coils L8, each of which has an inductance of about 0·4 henry. The cross-connecting condensers C6 are fixed mica condensers of 0·004 μF capacity each. The non-inductive grid and condenser leak resistances r_3 are 50,000 ohms each of eureka wire. The inducing coil L9 is connected in series with the right-hand grid resistance at the filament end of it when the switch S5 is to the right in the H.F. position.

The frequency adjustment on this multivibrator is made by adjustment of the filament brightness by means of the coarse and fine resistances, R_1, R_2 in the filament circuit. By this means the frequency can be brought within the narrow belt of synchronisation with the controlling current in L7. This belt is extremely narrow with regard to filament current in valves V5 and V6, on account of the rather rapid rate of change of frequency on this multivibrator with filament current. This necessitates a stud contact resistance R1 and a smoothly variable rheostat R2, of the kind used on potentiometers.

Adjustment of frequency of this multivibrator to synchronism is, however, not the only condition requisite to satisfactory operation. The characteristics of valves of different kinds differ to such an extent that although the frequency of operation can in general always be brought to the 20-kilocycles region the behaviour is not then always satisfactory. This is particularly the case with regard to the sub-harmonics produced by the modulation of the current in the controlling circuit, and doubtless also by the direct and capacity couplings of the low and high-frequency multivibrators.

In order to throw light on these effects some frequency characteristics were taken, using various types of valve on a separate high-frequency multivibrator. Curves were obtained connecting frequency and filament current for various anode voltages; observations of anode current were also made. Families of curves were drawn out for a number of different types of valve and for different specimens of the same valve. Pairs of valves had, of course, to be used in the test multivibrator, but observations of filament and anode currents were only made in the circuits of one of them.

In figs. 12 and 13 are shown curves for an ordinary R type (French) receiving valve. The curves of fig. 12 show frequency plotted against filament current for various constant anode battery voltages; the curves of fig. 13 are derived from those of fig. 12 and show the relation between frequency and anode battery voltage for various constant filament currents.

These curves are similar in general shape to those obtained in the case of a regenerative valve oscillatory circuit of the ordinary kind, as shown by W. H. ECCLES and J. H. VINCENT (14), also W. A. LEYSHON. (15). Since, however, the multivibrator is not an oscillatory system in the ordinary sense of the term, it might be expected to show

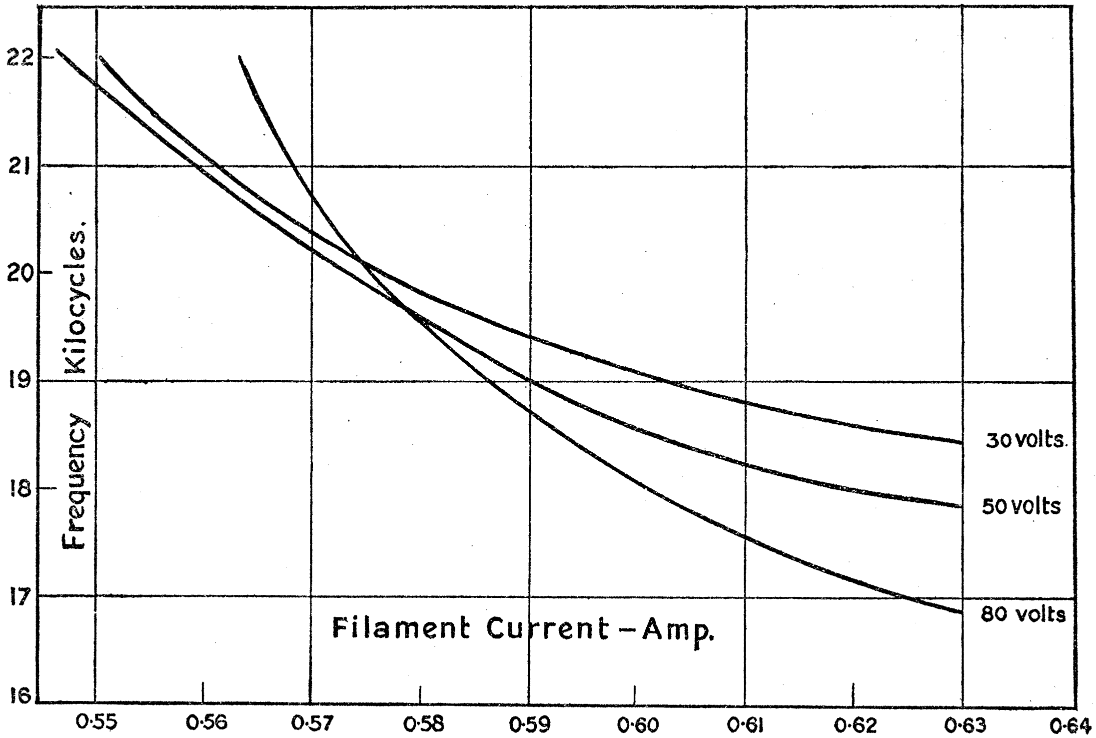

FIG. 12.—Variation of frequency of multivibrator with filament temperature.

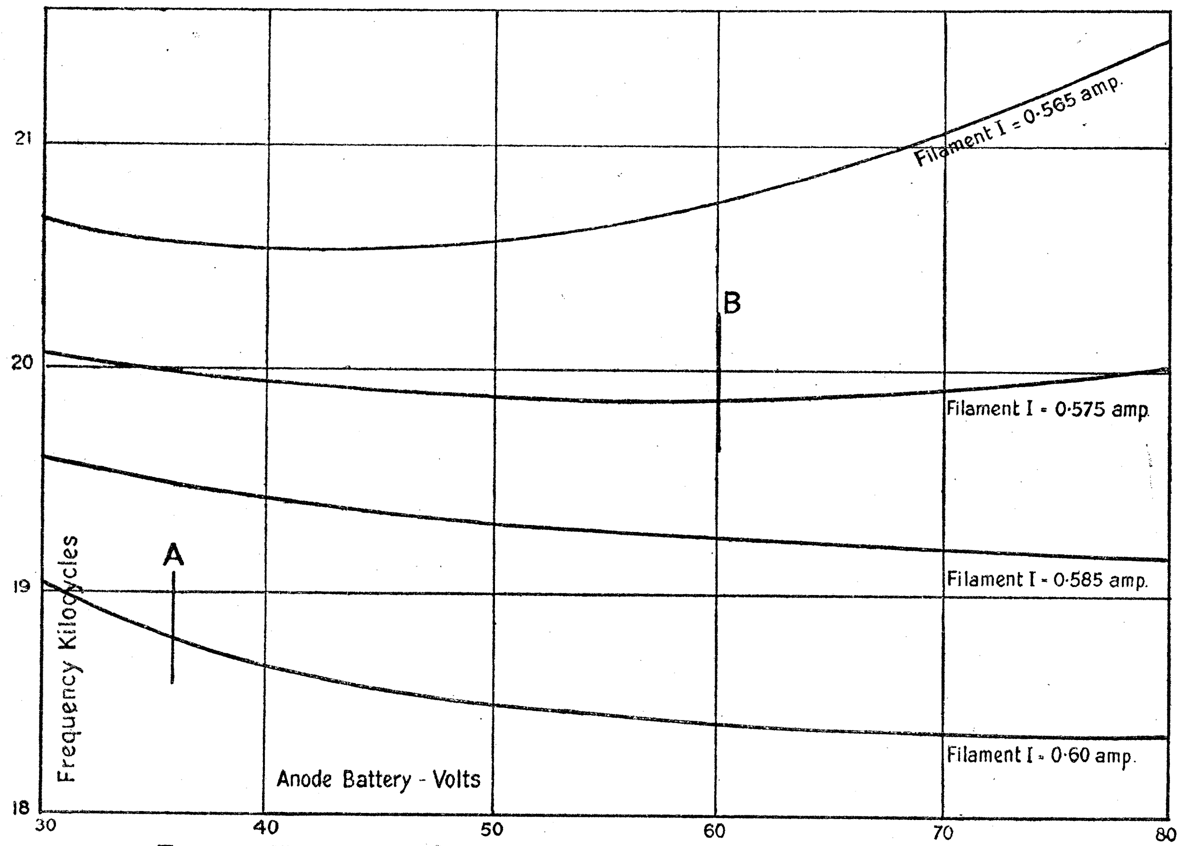

FIG. 13.—Variation of frequency of multivibrator with anode battery voltage.

reactions of a different nature to those obtaining in an ordinary regenerative oscillator when under the influence of varying conditions in the valve circuits. This is seen to be the case with respect to variations of frequency with variations of anode voltage. In the multivibrator case the curves connecting frequency and anode voltage can show a flat minimum in certain cases, whereas in an ordinary oscillatory system the frequency appears to always diminish with increasing anode voltage (14).

On examination of the curves of fig. 12, it was seen that the filament current adjustment for synchronisation would be less critical by working the multivibrator near the saturation region of the valve at a filament current in the neighbourhood of 0·53 amp. The experiment was therefore tried, on the compound multivibrator, of working the high-frequency multivibrator at the part of the curve where small frequency change occurred with change of filament current. In order to bring this part of the curve into the region of 20 kilocycles per second it was necessary to adjust the number of turns on the anode coils L8. This was done and synchronisation obtained : it was then found, as expected, that the adjustment of filament current did not need to be so precise in order to keep within the belt of synchronisation. Unfortunately, however, this condition carried with it the condition of strong modulation at 1,000 per second. This was so prominent that it was impossible to be certain which of the successive interference beats represented synchronisation with the main harmonic when an outside source was adjusted in frequency.

Further experimenting soon showed that the curves plotted on Fig. 13 gave the key to the behaviour of the high-frequency multivibrator. It was found to be always satisfactory when operating at such filament current and anode voltage that the curve connecting frequency and anode voltage was practically horizontal, indicating independence of frequency upon anode voltage over a considerable range of voltage. On adjustment of filament current and anode voltage to such values that the multivibrator was operating at a place such as A on curve IV of fig. 13, the sub-harmonics immediately became prominent. When, however, the conditions were as at B on curve II the behaviour was quite satisfactory.

This property of the multivibrator was found to hold true for five different kinds of valves. Plotting the curves of fig. 13 is the most satisfactory way of choosing the type of valve and of finding the suitable voltage and filament current for that type.

It may happen that none of the curves has a flat portion in the region of frequency of 20 kilocycles per second. In this case the whole family of curves may be shifted upwards or downwards in frequency by adjustment of the turns on the anode coils L8 of the high-frequency multivibrator. The change in frequency produced by this means is nearly proportional to the change in the number of turns. An ideal set of conditions would be that in which the family of curves in fig. 13 were flat and close together. No valve tried gave this condition, but it is possible that a compensating circuit might be so arranged as to bring about the condition of simultaneous invariability with anode battery voltage and filament current.

(b) *The Selector Circuit.*

As previously indicated, the object of this circuit is to select any desired harmonic. The two essential properties of the circuit are :—

(i) The proportion of the induced current at the desired frequency corresponding to the harmonic selected, to the currents induced by other harmonics should be as great as possible.

(ii) The selector circuit must be of such design and workmanship that the indication given by the variable part of it must leave no doubt as to which harmonic has been selected.

The circuit consists of a number of fixed inductance coils of small damping and such construction that they will remain invariable to within a few parts in a thousand in value. The capacity part of the circuit consists of a high-class variable air condenser, constructed to be as permanent in calibration as possible. Since harmonics up to the 120th can be selected, it is clear that the permanence of both inductance and capacity must be within about $\frac{1}{2}$ per cent. in order that the pointer of the variable condenser may never be more than, say, 0·25 of the space between two adjacent harmonics away from the engraved line representing a selected harmonic, when the condenser has been set to resonance at that harmonic.

The range of frequencies covered by each multivibrator is about twelve to one. This range requires a ratio of 150 to 1 between $C_{max.} \times L_{max.}$ and $C_{min.} \times L_{min.}$. A little consideration showed that this ratio could most satisfactorily be met by using three inductance coils for each multivibrator and by providing a ratio of maximum to minimum capacity on the condenser of about 7 to 1.

The design of the condenser has been largely determined by consideration of the shape of scales on which the harmonics are read. Since frequencies are read off on the scale the capacity will diminish as the readings increase. The capacity differences between successive harmonics become very small as the higher order harmonics are reached. This would cause, with the ordinary semi-circular shape of moving plate, a serious crowding together of the lines at the upper end. Thus, on such a condenser, if C_n is the capacity corresponding to harmonic "n" we then have

$$C_n = Q/n^2$$

where Q = a constant ; so that

$$\frac{dC_n}{dn} = -\frac{2Q}{n^3},$$

i.e., the angular interval between two successive harmonics varies inversely as n^3.

With a normal size of condenser scale the interval between the 100th and 101st harmonics would become only about 1·5 mm., an amount too small to ensure certainty in the event of the pointer receiving a small accidental displacement. It was, therefore, decided to design the condenser so as to give a uniform frequency scale. This requires a shape of vane somewhat like that for a uniform wave-length scale, or, again, like that for a direct reading decremeter. The equation to the shape of the vane is, however, quite different to either of these.

Condenser for uniform frequency scale.

The fundamental equations are :—

$$dn/d\theta = -q = \text{const.}, \quad (24)$$

where q = angular interval between successive harmonics.

$$C = B/n^2 \quad (25)$$

where

$$B = \frac{1}{4\pi^2 L}$$

and

$$dc/d\theta = Ar^2 \quad (26)$$

where r = radius of moving vane at angle θ, and A = constant determining the capacity of the condenser in suitable units, and thus governing the size of the condenser.

From (25) we have

$$\frac{dC}{dn} = -\frac{2B}{n^3} \quad (27)$$

which combined with (24) gives

$$\frac{dC}{d\theta} = +\frac{2Bq}{n^3} \quad (28)$$

Equating (26) and (28) gives

$$r^2 = +\frac{2qB}{An^3} \quad (29)$$

The integration of (24) may be written

$$n = q(\phi - \theta). \quad (30)$$

Substituting this value of "n" in (29) gives finally

$$r^2 = \frac{2B}{Aq^2} \cdot \frac{1}{(\phi - \theta)^3}. \quad (31)$$

A semi-circular opening must be made in the fixed-plate system to accommodate the spindle and washers of the moving-plate system. If the radius of this opening is "a" equation (31) becomes

$$r^2 - a^2 = \frac{2B}{Aq^2} \cdot \frac{1}{(\phi - \theta)^3} \quad (32)$$

The significance of the constant ϕ is that it is that angle (measured from the arbitrarily chosen axis where $\theta = 0$) at which the frequency "n" becomes zero; at this angle, therefore, "r" becomes infinite. Fig. 14 makes the equation quite clear.

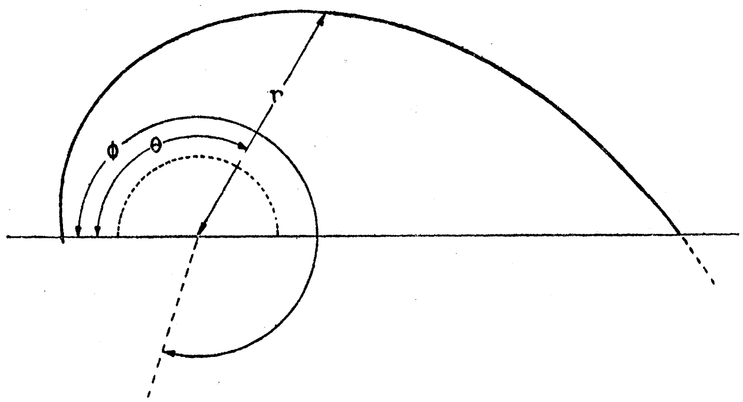

FIG. 14.—Shape of vane of condenser giving uniform frequency scale.

The figure is drawn to scale and shows the shape of vane as used on the condenser. Reverting again to equation (31), the expression for C in terms of θ becomes

$$C_\theta = C_0 + C_0 \frac{\theta(2\phi - \theta)}{(\phi - \theta)^2}, \quad \ldots \ldots \ldots \quad (33)$$

where C_0 = capacity when $\theta = 0$. The value of C_0 is quite arbitrary. If C_0 and ϕ are given definite values the condenser is determined. On the other hand, we may let $C_0 = C_{\text{min.}}$ and $C_{180} = C_{\text{max.}}$ have the values desired and so determine the spacing of the harmonics. In this case ϕ will become automatically determined and the shape of vane may be calculated.

In the actual condenser constructed C_0 is much larger than the value of the variable part of the condenser at the angle $\theta = 0$. A fixed air condenser is incorporated into the design and is so adjusted, after allowing for the self-capacities of the coils, that the required conditions are satisfied. The small difference between the self-capacities of the coils is of no consequence, since the departure from uniformity of the scales thereby produced is unnoticeable. Each scale has, of course, been marked off experimentally and is direct reading in kilocycles per second.

In order to reduce the effect of stray capacities to a minimum the condenser is shielded in a german-silver box; also, the coils—of disc form—are so arranged that the outer end of each is connected to the screen-connected terminal of the condenser. The only stray capacities which can affect the resonant setting of the condenser are those to the inner end of the coil and to the lead connecting it to the insulated terminal of the condenser. These parts are already largely screened by the outer part of the coil.

It has been necessary to consider in some detail these points regarding the selector circuit, in order to ensure certainty of indication of a selected harmonic. Ultimately, the only degree of permanence required is that the setting of the condenser to resonance at any desired harmonic shall not change adventitiously by an amount exceeding, say, one-quarter of the interval between two adjacent harmonics. In the worst case the capacity change between two adjacent harmonics, *i.e.*, 119th and 120th, is 1·6 per cent. of the total capacity. One quarter of this is 0·4 per cent. The condenser as constructed is not likely to change by this amount; in particular, at this region of its use the main part of the capacity is in the fixed portion of the condenser. Further down the scale (at, say, the 60th harmonic) the capacity is mainly in the variable part of the condenser, but in this region the change in capacity between adjacent harmonics is 3·3 per cent.

It is realised, of course, that when a harmonic has been selected and the condenser adjusted to give maximum loudness of beat tone in the amplifier telephone, even though the condenser pointer is then as much as 0·2 of a division from the line representing the selected harmonic, the frequency obtained is exactly that corresponding to the harmonic. So long as there is no doubt which line on the scale is referred to, there is no doubt regarding the frequency selected.

The chief mechanical features of the design of the condenser are the ribbed aluminium alloy top, to which everything is attached, the quartz washer insulators for holding the bank of fixed plates, and the accurately fitted bearings. The lower bearing does not put any axial constraint on the moving system. The tapering ends of the moving plates are stiffened by the provision of a small lug on each. A thin bolt with separating washers clamps and holds the tips of the vanes so that they are accurately spaced with respect to the fixed plates.

The Inductance Coils.

The design of these is beset with more uncertainties than the design of a condenser. There are so many variables, and the question of permanence has always to occupy such an important place, that they cannot be very simple in construction. The question of obtaining small decrement is mainly one of size and cost. In the present case it was decided to make the coils short in comparison with their diameter and to fix the outside diameter at 20 cms. for all the coils. It was hoped, under these conditions, that all the coils could be so designed that their logarithmic decrements would be in the neighbourhood of 0·01, and that for each coil the region of minimum decrement would fall within the working range of it. Formulas for the calculation of effective resistance of single layer and overwound coils at radio-frequencies have been developed by BUTTERWORTH (16) and more recently by FORTESCUE (17). These formulas are valuable as a general guide to the best proportions and number of strands to use for various coils. The formulas so far developed are not applicable to coils of disc form, or when wound of compact form and short axial dimensions.

There appears room for development of the formulas in this direction. The determination of the components of the magnetic field at any point in the section of the winding is extremely laborious when the coil is of compact form. The extrapolation required to the curves given by FORTESCUE is very uncertain when taken in the direction corresponding to coils having an axial length less than half the mean diameter.

The formulas developed by BUTTERWORTH assume the length and depth of winding to be small in comparison with the diameter; they assume also that the current is distributed uniformly over the section. In practice, however, the dimensions of the cross section are not small, and in the case of coils of less than about 100 total turns, the non-uniformity of current distribution over the cross-section of the winding is of importance.

There is thus some ambiguity as to what value to take for the term un in these formulas; this term may have very different values according to whether it is taken from the table for parallel wires or from the table for cylindrical coils, having the ratio of length to diameter chosen for the coil under calculation. It will be found, however, that, provided the correct number of strands is determined and the correct diameter of the individual strand approximated to, it will not then matter very much what the shape of cross-section is, so long as either the length or radial depth or both are not very far from 0·4 times the mean diameter.

If the section can be chosen with a ratio of its two dimensions of the order of 2·5 to 1, a good coil will result. In such a case the winding should proceed uniformly, in such a manner that the layers are many and short, in order that the distributed capacities from layer to layer may be across the minimum inductance. The self-capacity will then become as small as it is possible to make it with the design chosen, and the dielectric losses will also be a minimum.

The coils have been designed on the basis of these general considerations. The best number of strands and the best diameter of wire have not been available in some cases, and also the great desirability of using combs to locate the wires has caused the space factor of the winding to be much smaller than it should have been in the case of coils 1 and 2.

The coils have been constructed on built-up ebonite bobbins in such a manner that the spacing and location of the wires does not depend upon the wire itself. All the coils, except that for use in the region of frequency of 10^6 per second, have been wound with multiple-stranded silk-insulated wire. The size of the cable has been chosen as large as was consistent with the provision of space for strong enough separating and supporting combs to ensure permanent and accurate location of the wires. Measurements of effective resistance of a variety of types of stranded and solid wire coils of the inductance required by the last coil of the series indicated that a single layer solid copper-strip coil was the best form for frequencies in the region of 10^6 per second. The form adopted for this coil therefore was that of a flat helix of copper tape.

Measurements of the effective inductance and self-capacity of these coils were made over the range of frequencies at which they are used. The results are given in Table VII.

TABLE VII.

Coil.	Self-Inductance. Millihenries.	Self-Capacity. Micromicrofarads.
1	100·000	25
2	20·000	16
3	5·000	20
4	0·995	13
5	0·202	12
6	0·045	21

Measurements of the effective resistance were also made and the logarithmic decrement calculated for each coil at a number of frequencies. The results of these measurements are given in fig. 15 and show a number of points of interest. As mentioned above, it was hoped to obtain decrements as low as 0·01 for the coils, and at the same time it

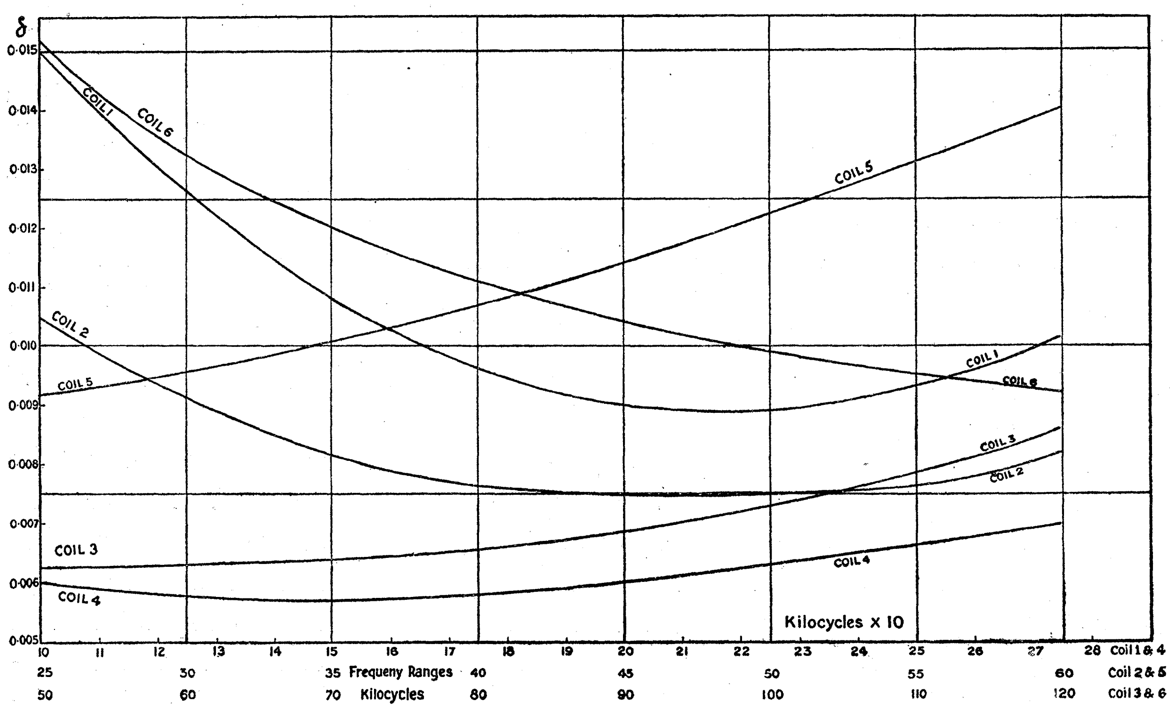

FIG. 15.—Log. dec. curves of inductance coils of selector circuit.

was hoped that the minimum decrement would fall within the working range. In the case of coils 1, 2, 3 and 4 these conditions have been realised. Coil 6 shows the typical decrement curve for the solid wire coil, but the curve of coil 5 is not understood. It is probable that a finer strand should have been used in the cable of which it was constructed. Since, however, the cable used already consisted of 81 strands of wire

0·12 mm. in diameter, it was not considered of sufficient consequence to obtain still more finely stranded cable.

The inductance coil of the selector circuit, and the coupling coils whereby the selector circuit is coupled to the multivibrator and to the amplifier, are attached at the back of the condenser in such a manner that the coupling coefficients will remain constant. The couplings and the number of turns on the various coupling coils have been chosen experimentally, so as to give the optimum effect in the amplifier telephones without causing any appreciable displacement of the resonant setting of the selector condenser.

(c) *The Detector Unit.*

This is mounted up as a separate piece of apparatus; it is of the tuned anode circuit type. The connections are shown in fig. 16. The inducing coil is connected to the grid-filament circuit of the first valve and has no tuning condenser. C1 L1 in the anode

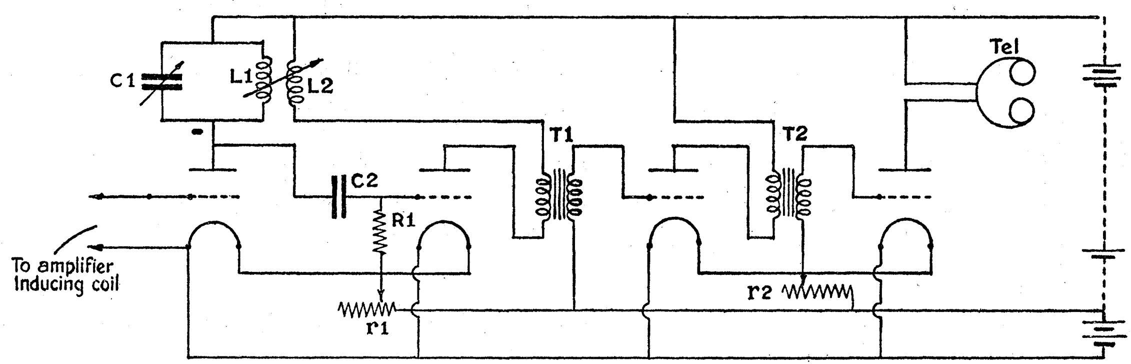

FIG. 16.—Connections of selective amplifier.

circuit of the first valve form the tuned anode impedance. C_1 consists of fixed and variable condensers reading up to 0·004 μF. L1 is the inductance, consisting of any one of a set of small-size coils. The range of coils is such that the circuit C1, L1 covers a range from 7 to 2,000 kilocycles per second. The coil L2 provides reaction for the purpose of increasing the selectivity of the apparatus and also to enable it to become self-generating on occasion. Two stages of low-frequency amplification are provided in the usual way.

A selective amplifier is essential when the multivibrator is being used with high-order harmonics, especially on the low-frequency multivibrator. By adjustment of the reaction the effective resistance of the circuit L1, C1 can be greatly reduced. In this condition the amplifier becomes extremely selective and requires very close setting of C1. When the correct adjustment has been found, the neighbouring harmonics to that one selected are rendered relatively weak, so that the interpretation of the interference tones heard is not too dependent on a trained ear.

A plan view of the apparatus as set up for use is shown in fig. 17 and indicates the general disposition of the parts.

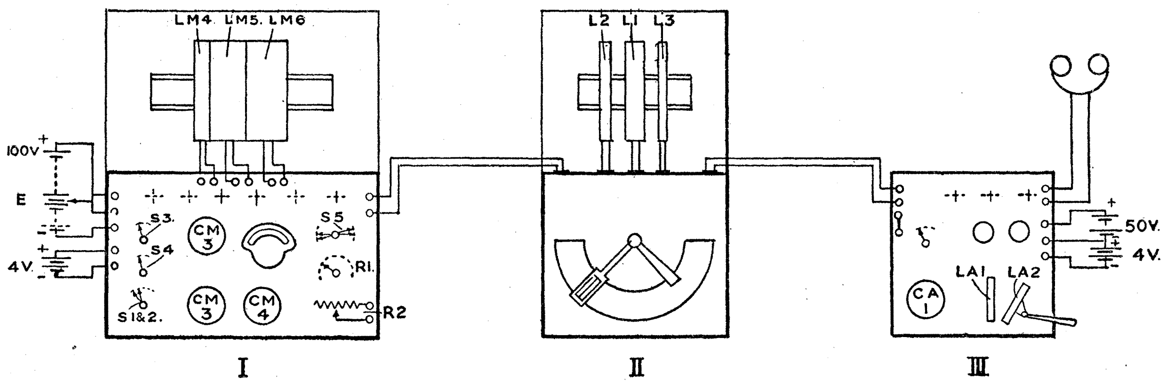

FIG. 17.—General disposition of parts of standard wave-meter.

7. METHODS OF USING THE MULTIVIBRATOR WAVE-METER.

The wave-meter described has been primarily designed as a fundamental laboratory standard of radio frequency ; as such, it has three main uses :—

(*a*) To provide a standard of frequency, whereby a steadily running local oscillator of small power may be set to any desired frequency for the purpose of calibrating wave-meters, and for any other purposes of measurement involving known frequencies.

(*b*) To enable standard waves of constant and known frequency to be transmitted from an aerial for the purpose of broadcasting the standard.

(*c*) To measure the frequency of sustained waves emitted by other transmitting stations.

With regard to (*a*) no difficulties whatever arise ; it is merely necessary to provide a valve oscillator of usual type and comply with the conditions for steadiness. The set in use at the Laboratory uses a transmitting valve of about 50 watts rating, run off an anode voltage of 400. The oscillator is set up on a neighbouring table and is provided with a set of interchangeable coils for the oscillatory circuit. An adjustable condenser consisting of fixed sections and a variable air condenser enable overlapping ranges of frequencies to be obtained with each coil. Four coils cover a range from 7 to 3,000 kilocycles per second. The steadiness of frequency is to 1 part in 100,000. Quite loud interference tones are produced in the amplifier telephone when adjusting this set to synchronisation with any desired frequency selected on the multivibrator. No particular coupling to the amplifier is required in general when using such a set ; the electrostatic coupling between the oscillator and the amplifier is sufficient on this size of generator. When, however, it is desired to directly calibrate a small oscillator wave-meter or other weak generator, it is necessary to provide a small amount of inductive coupling to the amplifier. This is easily accomplished by inserting a small coil of few turns in series with the amplifier inducing coil and bringing it near the inductive coil of the oscillator under test.

If it is desired to set or to measure a frequency which is considerably removed from a harmonic of the multivibrator, use is made of an accurately calibrated variable condenser in parallel with the main condenser of the oscillator. Fiducial points are found on this condenser corresponding to two or three neighbouring harmonics. Intermediate frequencies are then found by interpolation from a curve connecting variable condenser reading and frequency.

At the lower end of the range of the high-frequency multivibrator between, say, the 6th and 7th harmonics, corresponding to frequencies of 120 and 140 kilocycles per second, it will be seen that the interval is rather large for purposes of accurate interpolation. The gap can be shortened from either end by making use of successive sub-harmonics corresponding to frequencies of, say, 121, 122, 123, 124 and 139, 138, 137, 136 kilocycles per second. The gap can be reduced to about 16 per cent. by this means in the worst case. There is, however, another means of overcoming this trouble, and that is by working the high-frequency multivibrator at a fundamental frequency of 19 or at 21 kilocycles per second, instead of at 20; sufficient range in the tuning of the control circuit has been left to enable the 19th or 21st harmonic to be selected in this circuit from the low-frequency multivibrator for control purposes at these frequencies. Another series of frequencies is thus provided, such as 114, 133, 152 kilocycles per second from a 19 kilocycle per second fundamental; by the further use of sub-harmonics near these main harmonics the gaps may be reduced to about 2½ per cent. in frequency in the worst case.

These subsidiary scales are shown in relation to the main scales in fig. 18. Scales 4A and 5A refer to a fundamental of 19 kilocycles per second, and scales 4B and 5B refer to a fundamental of 21 kilocycles per second.

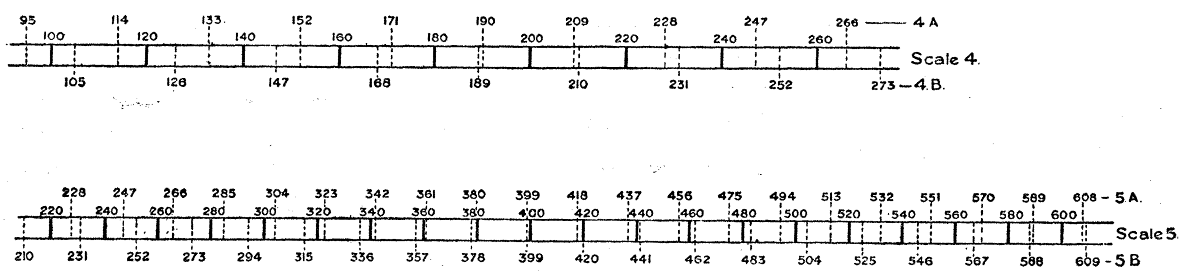

FIG. 18.—Subsidiary scales showing spacing of harmonics.

With regard to (*b*), the use of the multivibrator in connection with the transmission of standard waves from an aerial is really a special case of its use in connection with setting the frequency of a source. There are two convenient methods of carrying this out. The simplest of these is to make use of a local generator, such as that mentioned in (*a*), working on a suitable anode voltage. This generator is coupled by capacity coupling to the grid-filament circuit of the transmitting valve. This independent control on the power valve is a very satisfactory means of maintaining a transmission steady in current and frequency. In some actual tests made on an artificial aerial, about 70 watts of high-frequency energy was obtained from a 250-watt valve working on an anode voltage of 2,000. This represents about the maximum power that can be obtained from one valve on this voltage. By a cascade arrangement this power could be made to supply the grid-filament circuit of large valve or bank of valves.

This system of feeding the aerial requires adjustment of the small controlling oscillator and the steadiness obtainable depends upon the steadiness of this oscillator. There is, however, another means of producing steady high-frequency power, and that is to build-up from the multivibrator itself. For this purpose a cascade arrangement of valves is used, with tuned anode impedance in each anode circuit and with capacity coupling between anode and grids of the successive valves. This arrangement was tried; using three stages, enough power was available in the anode oscillatory circuit of the last valve to feed the grid circuit of the 400-volt valve set referred to above; this, in turn, can feed a 2,000-volt valve, thus giving high-frequency power actually controlled in frequency by the multivibrator. The current finally obtained contains a modulation at the tuning-fork frequency, but the amount of this causes no uncertainty regarding the frequency of the amplified harmonic, and this modulation is in some respects of value as it gives some character to the emitted wave.

With regard to (*c*), when measuring the frequency of transmission of a distant station the best procedure is to set a local oscillator to synchronisation, or preferably to a difference tone of 1,000 cycles per second, with the incoming signals, and then to measure the frequency of the local oscillator against the multivibrator by interpolation between two or more calibrated points on the open scale of a variable condenser forming a fractional part of the main capacity of the oscillator.

8. Precautions to be taken in using the Apparatus.

The frequencies given by the multivibrator are known with absolute certainty in terms of the controlling tuning-fork when rightly interpreted; a certain amount of skill in hearing is, however, needed, in order to understand what is actually being heard. For example, owing to the rectification of the amplifier a note of frequency of 1 kilocycle per second is always heard whenever a harmonic of the low-frequency multivibrator is selected and amplified; this note has nothing to do with the interference note heard against an external source. As a consequence, when the external source is varied to the extent of being almost 1 kilocycle per second different from the selected harmonic, a beating is heard between the two notes in the telephone, due respectively to the rectified modulated multivibrator current and to the true interference between multivibrator and external source. There is, therefore, a liability to set the external source to a frequency 1 kilocycle per second different to that intended. In order to ascertain that this mistake has not been made it is very desirable, after having set the external source to supposed synchronisation with a selected harmonic, to then select on the multivibrator selector circuit the two neighbouring harmonics on each side of that chosen. If the external source has been correctly set a loud note of 1 kilocycle per second will be heard when the selector condenser is set to the immediately adjacent harmonic on each side; or, again, a note of 2 kilocycles per second will be heard when the selector condenser is set to the next but one harmonic on each side.

Thus, suppose it is desired to set an external source to a frequency of 60 kilocycles per second. This harmonic is selected on the selector circuit and the amplifier tuned to it. The external source is now set to what is considered synchronism—there will not be complete silence in the telephone when this is done because of the rectified current referred to above, resulting in a note of 1 kilocycle per second—if now the selector circuit is set to 59 or 61 a loud sound at 1 kilocycle per second should be heard. When the selector circuit is set to 58 or 62 the fairly high note of 2 kilocycles per second will be heard and a fainter note of 1 kilocycle per second. If, however, a frequency of 61 kilocycles per second has been set in error on the external source, then, on setting the selector condenser to 61, a fainter note of 1 kilocycle per second will be heard, and on setting it to 59 a frequency of 2 kilocycles per second will be heard as well as the fainter one of 1 kilocycle per second. There is no doubt in judging these notes, so long as enough voltage is induced from the external source into the amplifier, unless the telephone has a natural resonance near to a frequency of 1 kilocycle per second. It may then happen that the 1 kilocycle per second component will drown the true beat tone of 2 kilocycles per second. In general, if Brown telephone receivers are used this effect will not occur, since these telephones have a fundamental natural resonance considerably above 1 kilocycle per second.

One other trouble likely to arise occurs when using the high-frequency multivibrator. Mention has been made of sub-harmonics at intervals of 1 kilocycle per second on each side of the main harmonic selected. These sometimes become prominent because the selector circuit or the amplifier tuning circuit have each not been set sufficiently carefully to the exact position of resonance. If, for example, the selector circuit condenser has been set slightly to one side of the true resonance position, then the sub-harmonics on that side will be reinforced and the true harmonic will be diminished. It is usually satisfactory to set the external source to a beat tone of about 800 or 1,500 per second and then to carefully tune selector circuit and amplifier condenser for maximum loudness of the note.

9. Conclusion.

The standard wave-meter here described is essentially a laboratory piece of equipment, but has been designed to be completely portable and not to require any elaborate setting up. In terms of the controlling tuning-fork it is believed that the wave-meter gives a series of frequencies of range and accuracy not hitherto obtained.

The fundamental accuracy can be checked at any time by measurement of the frequency of the tuning-fork by any satisfactory method. The accuracy of order of the harmonic can always be checked by the aid of an auxiliary heterodyne set, by making use of its fundamental and second harmonic.

Such a standard has other uses than that of measuring radio-frequencies. It may also be used in connection with measurement of inductance, capacity, and self-capacity of coils. Also very small changes in inductance and capacity may be measured by observing the changes in beat tone produced when the capacity or inductance under observation forms part of a valve-generator circuit.

There are possibilities also in the use of the apparatus, in a modified form, for the purpose of controlling the frequency of transmitting stations to an accuracy not hitherto attained.

In conclusion I wish to express my appreciation of the careful and accurate constructional work carried out by the staff of the workshop. Also to Mr. Martin of the Electrical Department for the skilful manner in which he has carried out many—sometimes tedious—observations with the apparatus.

My thanks are due also to Sir Joseph Petavel and to the members of the Committee of the Radio Research Board on Standards and the Propagation of Waves, established under the Department of Scientific and Industrial Research, under whose auspices the whole work has been carried out and who have shown much interest in the development of the standard.

A complete specification of the wave-meter, with detail working drawings, has been drawn up, copies of which are deposited at the National Physical Laboratory.

An Appendix giving full instructions in the setting up, use and adjustment of the wave-meter has also been prepared.

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